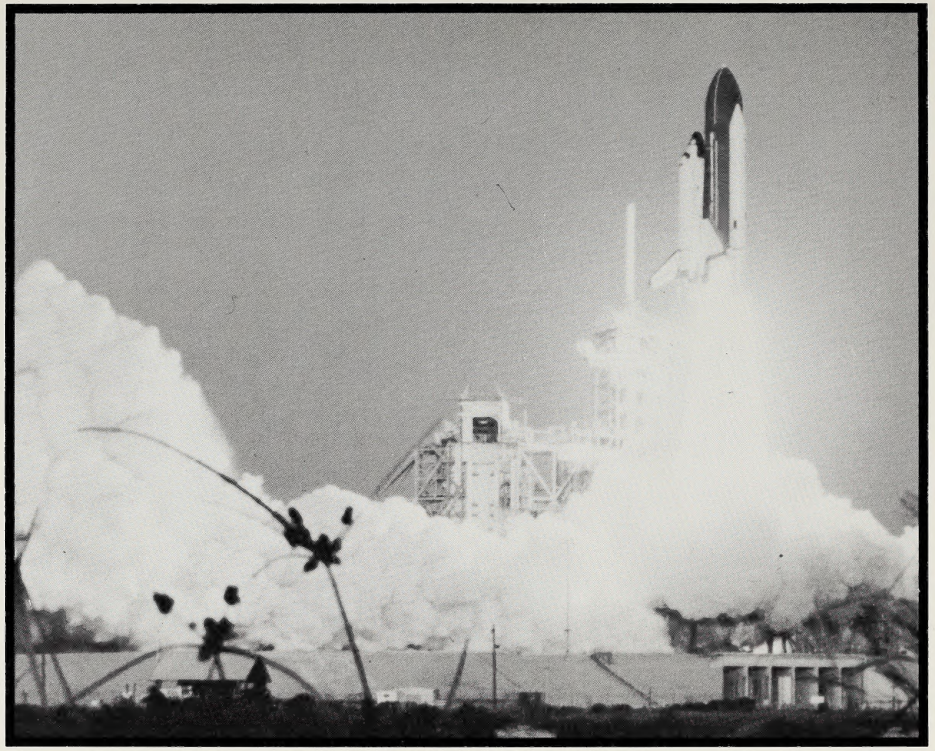
# MATHEMATICS



## Module 1: Problem Solving

Distance Learning

Alberta EDUCATION

University of Alberta Library
0 1620 3409851 5
CANADIANA
DEC 31 1993

# Mathematics 9

## Module 1

# PROBLEM SOLVING

Alberta
EDUCATION

**IT IS STRICTLY PROHIBITED TO COPY ANY PART OF THESE MATERIALS UNDER THE TERMS OF A LICENCE FROM A COLLECTIVE OR A LICENSING BODY.**

| This document is intended for | |
|---|---|
| Students | ✓ |
| Teachers (Mathematics 9) | ✓ |
| Administrators | |
| Parents | |
| General Public | |
| Other | |

UNIVERSITY LIBRARY
UNIVERSITY OF ALBERTA

Mathematics 9
Student Module
Module 1
Problem Solving
Alberta Distance Learning Centre
ISBN No. 0-7741-0844-4

Cover Photo: NASA

**ALL RIGHTS RESERVED**

Copyright © 1993, the Crown in Right of Alberta, as represented by the Minister of Education, Alberta Education, 11160 Jasper Avenue, Edmonton, Alberta, T5K 0L2.
All rights reserved. Additional copies may be obtained from the Learning Resources Distributing Centre.

No part of this courseware may be reproduced in any form, including photocopying (unless otherwise indicated), without the written permission of Alberta Education.

Every effort has been made both to provide proper acknowledgement of the original source and to comply with copyright law. If cases are identified where this has not been done, please notify Alberta Education so appropriate corrective action can be taken.

*Welcome to Module 1!*

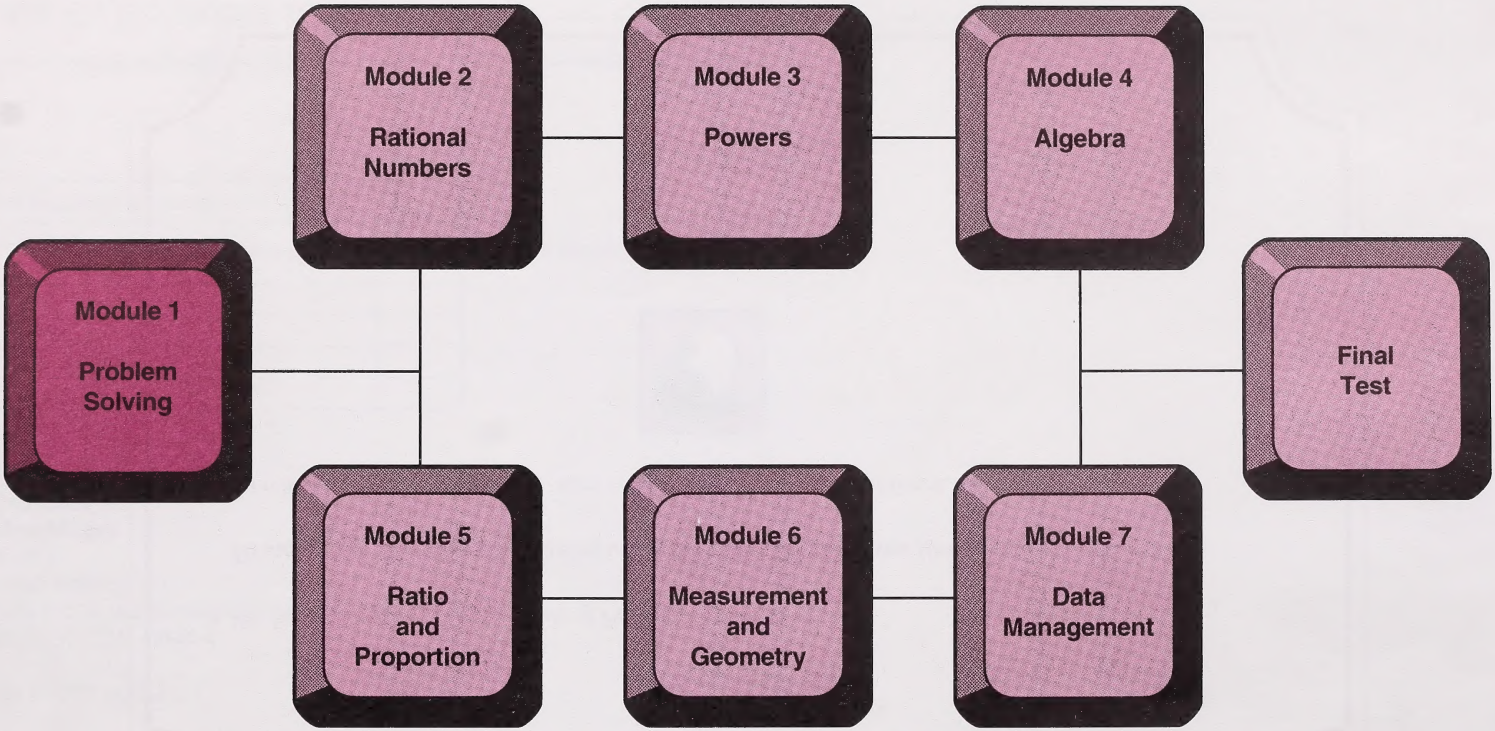
*We hope you'll enjoy your study of* ***Problem Solving****.*

*To make your learning a bit easier, a teacher will help guide you through the material.*

*So whenever you see this icon, turn on your audiocassette and listen.*

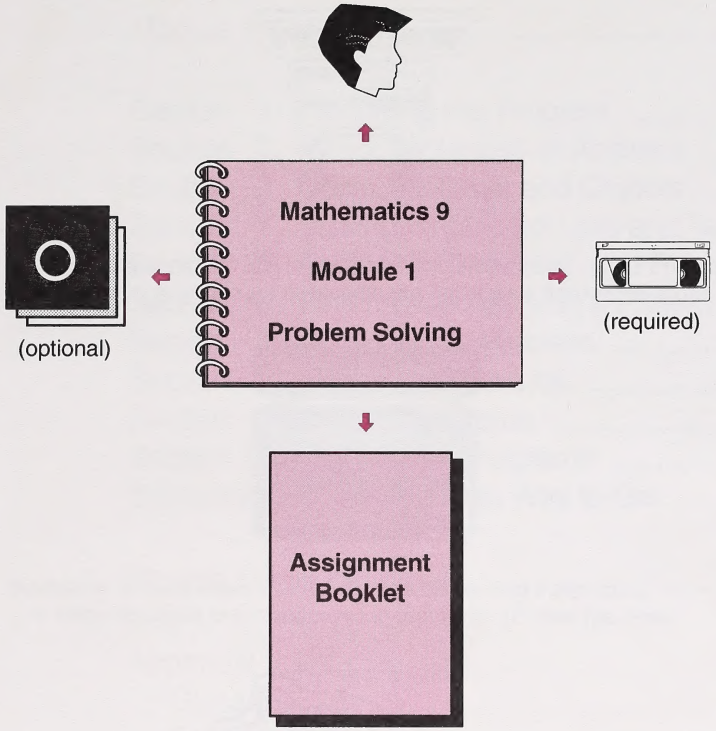
*Turn the audiocassette on now to begin.*

## Course Overview




Mathematics 9 has seven modules and a final supervised test.

## Module 1 Components




This booklet will give you instruction and practice in learning mathematical skills and words. It will also direct you to the other components of the module: the companion audiocassette, the videocassettes, the computer software, and the Assignment Booklet.

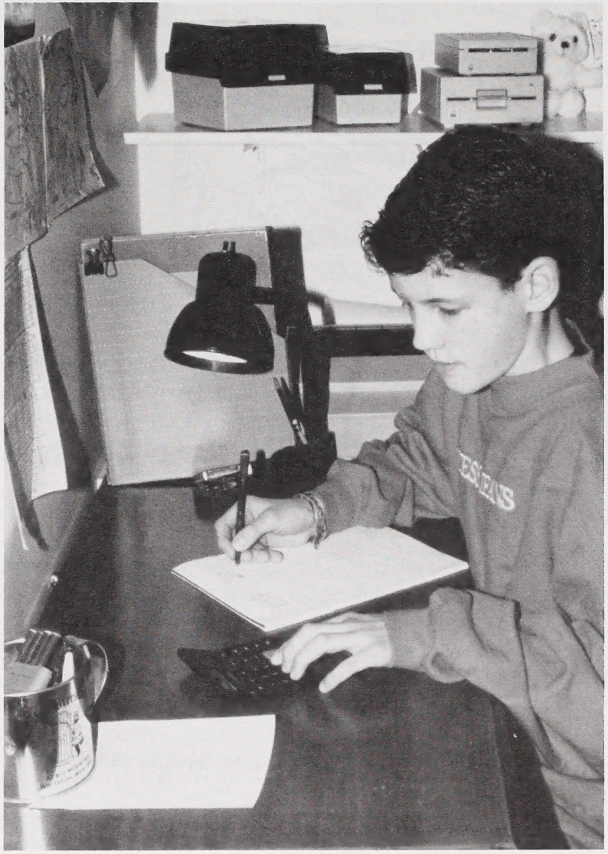
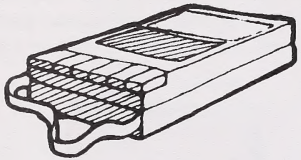


Photo Search Ltd.

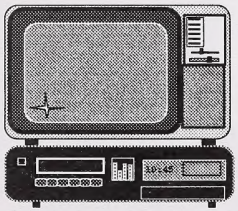
There are no response spaces provided in this Student Module Booklet. This means that you will need to use your own paper for your responses. You should keep your response pages in a binder so that you can refer to them when you are reviewing or studying.

## Optional Equipment

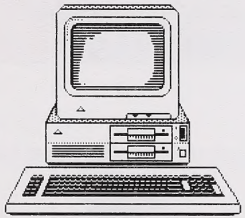
The companion audiocassette for this module is optional. If you decide to listen to it, you will need an audiocassette player.



The video activities in this module are required. To view the video programs, you will need a videocassette player and a television.



The computer activities in this module are optional. If you decide to do the computer activities, you will need an Apple computer.



## Required Equipment

You will need a geometry set and a calculator for this module. The calculator should have an $a\frac{b}{c}$ key so that you can perform operations on fractions.

## Evaluation

Your mark on this module will be determined by your work in the Assignment Booklet.

Your responses to the questions in this Student Module Booklet are not to be submitted for a grade. However, it is important that you work through the activities carefully before attempting the questions in the Assignment Booklet. This will help you achieve a greater degree of success in your studies.

Discuss how the module will be evaluated with your learning facilitator.

## Time Management

Decide how long you will need to complete the module. Your learning facilitator will help you plan a schedule.

# CONTENTS AT A GLANCE


Module Introduction ..... 1

Section 1: Identifying the Problem ..... 3
Section 2: Reasonableness of Answers ..... 5
Section 3: Using Sketches and Objects ..... 7
Section 4: Making Organized Lists and Tables ..... 10
Section 5: Guessing, Checking, and Revising ..... 13
Section 6: Finding and Applying a Pattern ..... 16
Section 7: Simplifying a Problem ..... 20
Section 8: Working Backwards ..... 23
Section 9: Using Elimination ..... 28
Section 10: Using Venn Diagrams ..... 32
Section 11: More than One Way to Go ..... 37

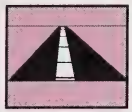
Module Conclusion ..... 41

Appendix ..... 42
Glossary ..... 43
Suggested Answers ..... 43

Digitized by the Internet Archive
in 2016 with funding from
University of Alberta Libraries

https://archive.org/details/mathematics901albe

# MODULE INTRODUCTION



## What Lies Ahead

In the module introduction you will preview Module 1.

## Working Together

One of the main goals of this mathematics course is to improve your problem-solving skills. You will need these problem-solving skills many times in your lifetime.

Like any skill, the skill of problem solving must be developed.

This module will give you hints about problem solving. You will have many opportunities to practise solving problems in this module and in the other modules of the course.

Since this module focuses on problem solving, it is important that you understand what a problem is. A **problem** is a task for which the method of finding the answer (as well as the answer) is not immediately known.

Problems may or may not involve computation (adding, subtracting, multiplying, and dividing). Some problems are realistic, others are puzzles.

## The Four-stage Process

There are four stages that can be used to solve any problem.

### Understanding the Problem

In this stage you should realize that you are dealing with a problem. There are various reasons for this.

- You may not know the meanings of all the words.
- You may not understand the situation in the problem.
- You may be confused by unnecessary information.

Strategies for understanding the problem will be discussed in Section 1.

At this stage you should also think about the problem and make an estimate of what the answer should be. This will help you arrive at a reasonable answer. Section 2 will discuss reasonableness of answers.

### Developing a Plan

This is where you should decide on the plan of action that you are going to take to solve the problem.

You may consider the following strategies:

- drawing a sketch
- using objects
- making an organized list
- guessing, checking, and revising
- finding and applying a pattern
- making a table
- simplifying the problem
- working backwards
- using elimination
- using Venn diagrams

Each of these strategies will be discussed in greater detail later in this module.

## Trying the Plan

In this stage you should try the plan and see if it works.

If you are doing calculations, be sure to work carefully and record your progress. You are encouraged to use a calculator to help with your calculations.

**Note:** While trying the plan, you should monitor your progress in order to determine if your plan will lead to a solution. You may find that the plan will not produce a solution, in which case a new plan will have to be developed.

## Looking Back

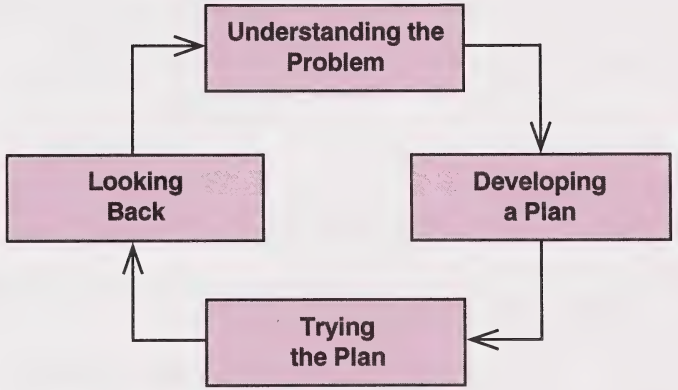
In this stage you should look back at the problem and compare your answer to the estimate you made in the first stage. Restate the problem using your answer to test it and ask yourself these questions: "Did my plan work? Is my answer reasonable?"

If you did not arrive at an answer, or if your answer is unreasonable, you may need to repeat one or more of the earlier stages. You might have made errors while trying your plan. Perhaps another approach would work better.

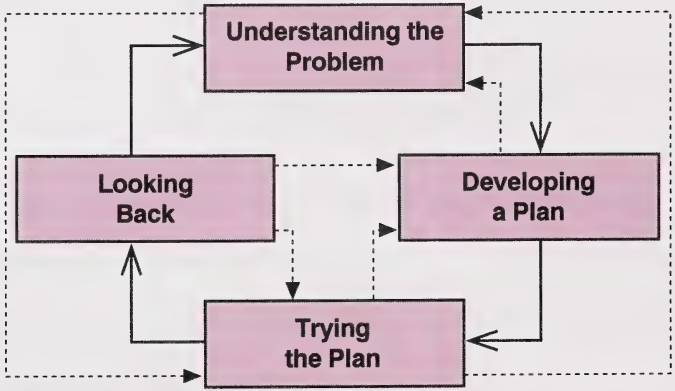
Section 2 discusses reasonableness of answers. If you arrived at a reasonable answer, you may wish to consider other possible answers or other ways to solve the problem. Section 11 discusses alternative strategies to solve the same problem.

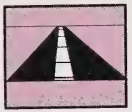
**Note:** People sometimes forget the looking-back stage and think they are done the problem when they find an answer. Do not overlook this important stage.

A problem can usually be approached in the order outlined in the following diagram.




If you encounter difficulties in your original approach, or if you realize that another approach will have better results, you may need to return to an earlier stage or use the stages in a different sequence.





## What Lies Ahead

In this section you will learn these skills.

- identifying the essential elements of a problem (what you know and what you need to know)
- ignoring unnecessary details
- changing the setting of a problem to help understand the problem
- restating the problem in your own words

## Working Together

Do you remember the four-stage problem-solving process? The first stage is understanding the problem.

This section will deal with techniques that you can use to help you understand what you are being asked to discover in a problem.

## Video Activity

Watch the program *MATH WORKS: Identifying the Problem* (Agency for Instructional Television). The program has three stories and a real-life application.

## Program Summary

### Story 1

Jenny and Arlene want to give their grandmother a jewellery box for her eightieth birthday. The girls have saved some money and they want to figure out how much more they need.

Arlene thinks that "more" means that they need to use addition to solve the problem. She figures out the problem, but gets an incorrect answer. Jenny realizes that the answer is unreasonable, so they discuss what they already know and what else they need to know. Jenny includes many unneccessary details in the list of what they need to know, but Arlene points out that these should be ignored. Their discussion helps them understand that "more" actually means that they need to use subtraction to solve the problem correctly.

### Story 2

Andy and Paul are doing their homework. They have been given a problem about a farmer, some hens, and some eggs. The problem seems difficult to them because they are unfamiliar with farm life.

To help them understand, they decide to change the setting of the problem. When Paul substitutes bicycles and wheels for hens and eggs, he is able to understand the new problem. This gives him ideas about how to solve the original problem.

### Story 3

Mark and Sharon are shopping for wieners in the grocery store. They want to figure out which package of wieners to buy – a package of ten for $1.98 or a package of six for $1.08. They are confused, so they decide to restate the problem in their own words to make it more understandable.

### Real-life Application

An interior designer states that the best way to avoid problems in a detailed project is to ignore unnecessary information.

## Practice Activity

**1.** Cross out the unnecessary details in the following problems and then restate the problems in your own words.

**a.** Marcel is working very hard to earn enough money to buy a new $425 bicycle. He gets paid $14 per week for babysitting two hours on Thursday and five hours on Saturday. As well, he earns $20 per week for delivering newspapers between 6:30 and 7:30 a.m. every morning. He also gets paid $5 every Wednesday for spending 1.5 h delivering weekly advertising flyers. For how many weeks will Marcel have to work so that he can buy his new bicycle?

**b.** A plane leaves Vancouver at 7:30 a.m. and arrives in Toronto at 12:00 noon Vancouver time. It flies a distance of 3400 km at a cruising speed of 950 km/h. When it is 6:00 a.m. in Vancouver, it is 9:00 a.m. in Toronto. What time is it in Toronto when the plane lands?

**2.** Change the setting of the following problems.

**a.** A numismatist bought a rare sovereign for $30, sold it for $40, bought it back for $60, and sold it again for $70. How much money did the numismatist make or lose in buying and selling this sovereign?

**b.** Martin spent twice as much on shantung as he spent on chambray. If he spent $30 on chambray, how much did he spend on shantung?

Turn to the Appendix to check your answers.

## What Lies Ahead

In this section you will learn these skills.

- estimating an answer
- determining if an answer is reasonable

## Working Together

When solving problems, it is important to spend time estimating the answer and looking back to see if the answer is reasonable.

### Video Activity

Watch the video *SOLVE IT: Reasonableness of Answers* (Agency for Instructional Television). This program has three stories and a real-life situation. As you watch the program, look for situations in which the characters need to check the reasonableness of their answers. Be sure to notice the methods they use to do this.

## Program Summary

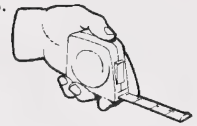
### Story 1

A teacher poses two math problems to the class. Toby answers incorrectly because he does not estimate the answer and he does not think about the reasonableness of his answer.

Ebony first makes a mental picture and thinks about the problem situation in order to imagine what the answer will be like. Afterwards, she thinks about the reasonableness of her answer by comparing her answer to her estimate.

### Story 2

Lisa and Bobby are fencing their front yard. They must fence 6 m of the yard, and the fence comes in 2-m wide sections. Bobby figures out the problem incorrectly because he does not think about the problem situation and he overlooks thinking about the reasonableness of his answer.



Lisa realizes that Bobby's answer is unreasonable because she has made a mental picture of the situation.

### Story 3

Toby and Gina are helping the owner of a sporting goods store. Their job is to find the total value of the bicycles in the store. Toby makes an estimate of the cost of the bicycles before he begins calculating. This helps him check the reasonableness of his answer afterwards.

### Real-life Situation

A pediatrician (children's doctor) explains how she uses problem solving in her job. First she tries to understand the problem. Then she makes a diagnosis, prescribes a treatment, and checks the results. If the treatment is effective, she has solved the problem. If the treatment is not effective, she must think about the problem again and plan another treatment.

## Practice Activity

For Questions 1 to 3, estimate the answers and then tell which of the provided answers is the most reasonable and explain why.

1. If a person walks 1 km, how many steps are taken?

   **A.** 25 **B.** 100 **C.** 2000

2. A hamburger patty has a mass of about 100 g. How much ground beef is needed if 25 people eat two hamburgers each?

   **A.** 5 kg **B.** 50 kg **C.** 500 kg

3. If you use 2 mL of toothpaste each time you brush your teeth (three times a day), about how many weeks will it take to use up a 100-mL tube of toothpaste?

   **A.** 2 weeks **B.** 6 weeks **C.** 12 weeks

For Questions 4 to 6, calculate the answers. Then tell which of the provided answers is the most reasonable and explain why.

4. The students in the ninth-grade class are going on a field trip. If there are 29 students and four students can fit in each car, how many cars are needed?

   **A.** 7 **B.** 1 **C.** 7.25 **D.** 8

5. Sorena has 29 cassette tapes. She wants to arrange them in a box which will hold four tapes in each row. How many tapes will she have in the last row?

   **A.** 7 **B.** 1 **C.** 7.25 **D.** 8

6. Patrick's jazz band rehearsed a total of 29 h in the last four days before the concert. If they spaced their rehearsals equally over the four days, how many hours did they practise each day?

   **A.** 7 **B.** 1 **C.** 7.25 **D.** 8

7. The local burger shop sold 29 hamburgers in 4 min. About how many hamburgers were sold per minute?

   **A.** 7 **B.** 1 **C.** 7.25 **D.** 8

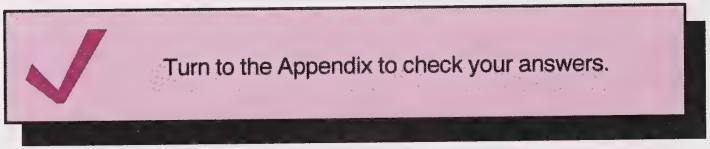
Solve Questions 8 to 10. Be sure to estimate your answer first. Then compare your calculated answer to the estimate to determine if the calculated answer is reasonable.

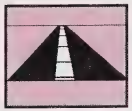
8. Shamaz would like to buy four items worth $95.82, $53.21, $9.58, and $64.49. Will he be able to purchase all the items if he has $200 to spend?

9. A service club is selling raffle tickets on a new car. If each member is responsible for selling 18 tickets, how many tickets will be sold by 182 members?

10. The glee club sold buttons for $0.25 each. If they raised $350 from the sale, how many buttons did they sell?

11. A shortbox truck costs $9895. What would be the amount of each monthly payment, excluding interest, for buying this truck over one year?





## What Lies Ahead

In this section you will learn these skills.

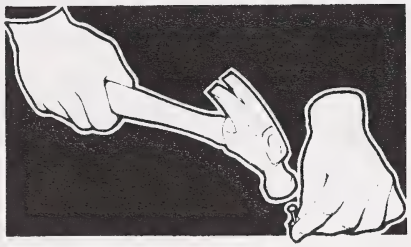
- using sketches to organize information
- using objects to act out a problem or organize information

## Working Together

There are many strategies you can use to help you understand a problem. This section deals with using objects and drawing sketches to help solve problems.

### Example 1

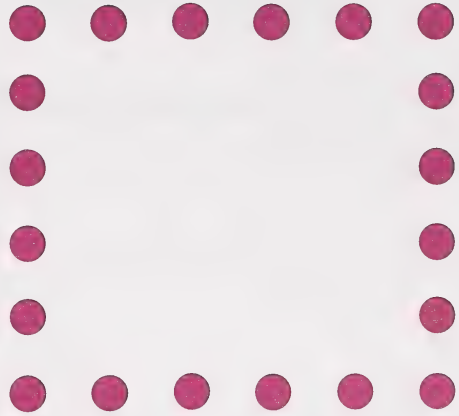
Twenty posts are used to enclose a square lot. If the posts are placed 3 m apart, what is the length of one side of the lot?



### Solution

You could use objects to represent the posts or you could make a sketch. The advantage of using objects to represent the posts is that you can easily rearrange them.

It does not matter if you use objects or if you make a sketch – the result will be the same. You will find that the posts must be arranged as shown in the following diagram.



From the diagram you can see that there are six posts on one side of the square, but there are five spaces between the posts. If the posts are placed 3 m apart, the length of one side of the lot can easily be calculated.

$$5 \times 3 = 15$$

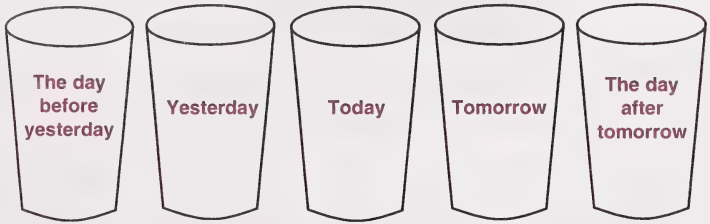
The length of one side of the lot is 15 m.

### Example 2

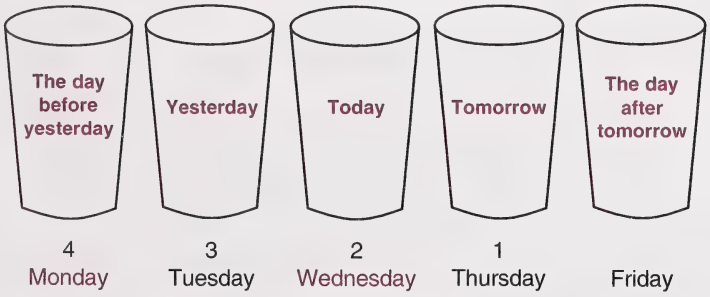
Four days before the day after tomorrow was Monday. What day of the week is it today?

### Solution

You could use paper cups to represent the days. The cups could be labelled as shown.




Begin at the cup representing the day after tomorrow and count back four days.




You will find that Monday was the day before yesterday. You will also discover that today is Wednesday.

You could make a sketch instead of using objects.

| | | |
|---|---|---|
| 4 | (Monday) | The day before yesterday |
| 3 | (Tuesday) | Yesterday |
| 2 | (Wednesday) | Today |
| 1 | (Thursday) | Tomorrow |
| | (Friday) | The day after tomorrow |

Today is Wednesday.

## Practice Activity

### Computer Alternative

1. You'll need Disk 1 of *MATH STRATEGIES: Solving Problems* (SRA) to do this problem.

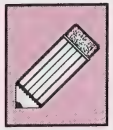
   From the chapter menu choose "5 Using Models." Then choose "1 Museum Trip."

   After you've read the problem, decide how much help you want. Follow the directions on the screen.

   **Note:** Whenever a smiling face appears, the computer stops. To make it continue, press RETURN.

2. You'll need Disk 1 of *MATH STRATEGIES: Solving Problems* (SRA) to do this problem.

   From the chapter menu choose "5 Using Models." Then choose "Measuring Puzzle." Complete as many measuring puzzles as you want.



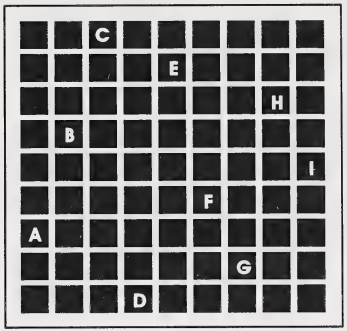
**Print Alternative**

3. Chris' office is 8 km from his home. Yesterday morning he drove 2 km before he realized that he had forgotten his briefcase. He returned home to get his briefcase and then drove to his office. At noon Chris drove 4 km to run some errands. He then went back to the office for the rest of the afternoon. At the end of the day he drove straight home. How far did he drive yesterday?

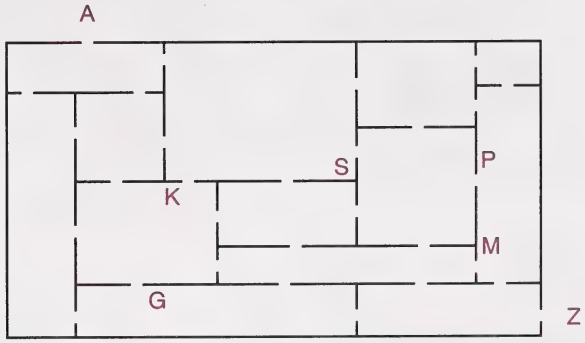
4. A fox, a duck, and a bag of corn are to be taken across the river by a farmer. Only the farmer and one other animal or item can fit in the boat. If left alone, the fox will eat the duck or the duck will eat the corn. How can the farmer get them across the river safely?

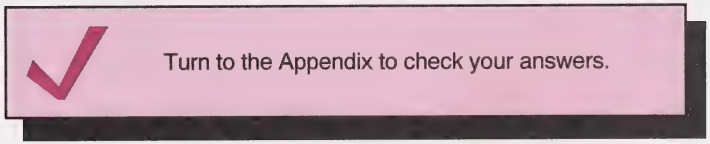
5. How would you plant ten shrubs in five rows of four each?

6. Nine students are seated as shown in the diagram to the right. When the teacher leaves the room, three students move one seat horizontally, vertically, or diagonally from their original seats. Since no two students are seated in the same row, column, or diagonal, the teacher doesn't notice the changes. Which three students have moved and to where?




7. A class visited the art museum on a field trip. They entered through doorway A and departed through doorway Z. They passed through each doorway only once, except for one doorway, which they did not pass through at all. Which of the lettered doorways did they not pass through?



# MAKING ORGANIZED LISTS AND TABLES

## What Lies Ahead

In this section you will learn these skills.

- making organized lists to solve problems
- using tree diagrams to make organized lists
- making tables to organize information

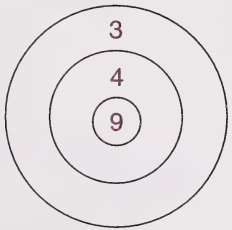
## Working Together

Some problems have a lot of data (facts and information).

There are several ways you can organize data to help solve a problem.

### Example 1

Julio and his friends like playing a game called Bull's Eye with lawn darts. They draw a target in the dirt and assign values to the three regions of the target.




If Julio throws three darts and all three hit the target, how many different point totals are possible?

### Solution

You can make an **organized list** of all the possibilities. The organized list arranges the point totals from the highest possible score to the lowest possible score.

$9+9+9=27$
$9+9+4=22$
$9+9+3=21$
$9+4+4=17$
$9+4+3=16$
$9+3+3=15$
$4+4+4=12$
$4+4+3=11$
$4+3+3=10$
$3+3+3=9$

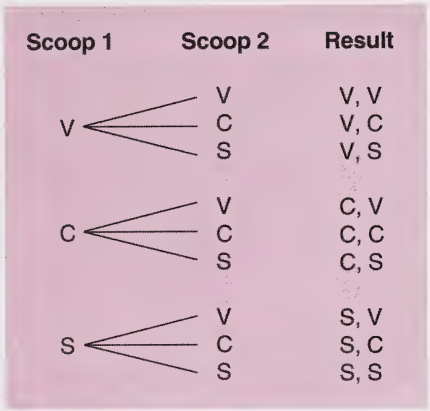
There are ten possible point totals.

### Example 2

At an ice cream parlour there are three flavours of ice cream. How many different double-scoop ice-cream cones are possible? **Hint:** A chocolate-vanilla cone is different from a vanilla-chocolate cone. Order makes a difference.

### Solution

You can draw a **tree diagram** to help you make an organized list. Use V for vanilla, C for chocolate, and S for strawberry.




Nine different double-scoop ice-cream cones are possible.

### Example 3

There are eight vehicles in a parking lot. Some are motor bikes and some are cars. Arana counted 26 wheels altogether. How many of the vehicles are motor bikes and how many are cars? **Hint:** Motor bikes have two wheels and cars have four wheels.

### Solution

You can arrange the data in a table.

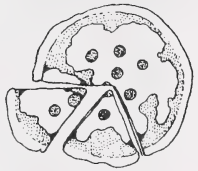
| Number of motor bikes | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| Number of cars | 7 | 6 | 5 | 4 | 3 | 2 | 1 |
| Number of wheels | 30 | 28 | 26 | 24 | 22 | 20 | 18 |

The table shows that there are three motor bikes and five cars in the parking lot.

## Practice Activity

1. Stacy has four different posters of horses: a Morgan, a Tennesee Walking Horse, an Arabian, and a Lippizaner stallion. How many ways could she display these posters in a straight row on her bedroom wall?

2. Peerless Pizza offers a choice of four different sizes, six different toppings, and either mozzarella or cheddar cheese. How many different pizza combinations are possible?



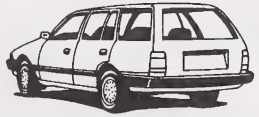
3. To determine their overall standing at the end of the season, all the downhill skiers who competed were awarded five points for each win, three points for each second place, and two points for each third place finish. If Aviva accumulated 15 points, what possible combinations of first, second, and third places could she have achieved over the season?

4. If you have a supply of 3¢ stamps and 8¢ stamps, how many ways can you make a total of $1.47 in stamps?

5. How many different ways can a football team score 35 points if it scores only field goals (three points) and converted touchdowns (seven points)?

6. Steven works at his mother's store after school. He earns 20¢ for each customer that he assists and 35¢ for each carton that he unpacks. One day he did 19 jobs and earned $4.85. How many customers did he assist that day and how many cartons did he unpack?

7. Station wagons have five doors and hatchbacks have three doors. At the Will U Buy Used Car Lot there are 16 of these vehicles with a total of 62 doors. How many station wagons and hatchbacks are there?



Turn to the Appendix to check your answers.

## Did You Know?

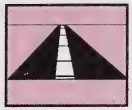
### Bhaskara

Bhaskara, who lived in India around A.D. 1150, wrote a book full of puzzles and stories.

One problem describes the god Sambu, who had ten hands. These hands held a rope, a book, a snake, a drum, a skull, a spear with three points, a bed, a dagger, a bow, and an arrow. Sambu wants to know how many ways he can pick up these things[1].

[1] National Council of Teachers of Mathematics for the excerpt from *Mathematical History.*

# GUESSING, CHECKING, AND REVISING



## What Lies Ahead

In this section you will learn these skills.

- solving problems by guessing, checking, and revising your answer
- organizing your guesses in a table

## Working Together

This section deals with another useful strategy called guessing, checking, and revising.

This strategy involves using what you know about the situation to make a reasonable guess, checking the guess against the conditions in the problem, and using the findings of the test to revise the guess.

### Video Activity

Watch *SOLVE IT: Guess-Check-Revise* (Agency for Instructional Television) to discover how the guessing, checking, and revising strategy can be used to solve problems.

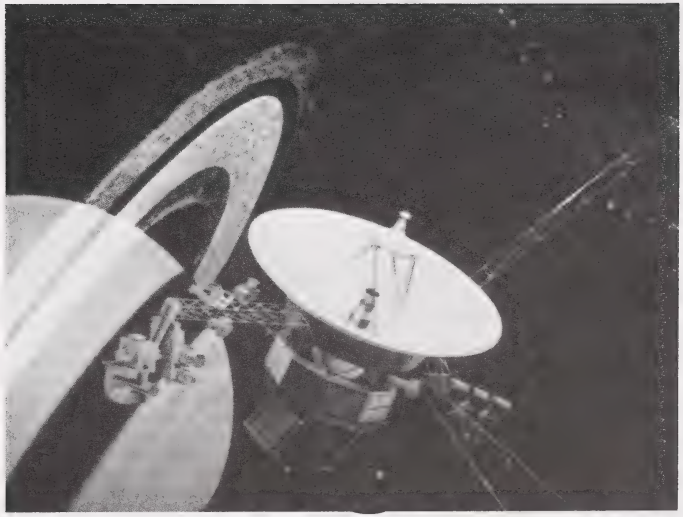
## Program Summary

### Story

Claude and Keisa's family has moved into a new apartment and they have two problems. They need to move a large chair through a narrow doorway and they need to hook up the cables to the VCR. Claude and Keisa solve these problems through a process of guessing, encountering difficulty, making better guesses on the basis of what they have learned, and guessing again.

### Real-life Application

A project planner explains how the guessing, checking, and revising strategy was an essential part of planning the Voyager space missions.



NASA

## Organizing Your Guesses

Many mathematical problems can also be solved using the guessing, checking, and revising strategy.

### Example

Two boys are talking about their baseball cards.

Matt said to Jon, "Give me one of your cards and I will have as many as you have."

Jon said to Matt, "Give me one of your cards and I will have twice as many as you have."

How many cards does each boy have?

### Solution

To help you understand the problem, you could use objects to represent the cards and act out the problem.

First make a reasonable guess about how many cards each boy has. Then act out the problem using objects to check the guess.

- What would happen if Jon gave Matt a card? The first condition to be met is that they must have an equal number of cards after the exchange.
- What would happen if Matt gave Jon a card? The second condition to be met is that Jon will have twice as many cards as Matt after the exchange.

If your guess does not fit the conditions, revise your guess and repeat the process. It is helpful to organize your guesses in a table.

| | Matt's Cards | Jon's Cards | Tests | |
|---|---|---|---|---|
| | | | First Condition | Second Condition |
| Guess 1 | 1 | 3 | $2 = 2$ | $4 \neq 2 \times 0$ |
| Guess 2 | 2 | 4 | $3 = 3$ | $5 \neq 2 \times 1$ |
| Guess 3 | 3 | 5 | $4 = 4$ | $6 \neq 2 \times 2$ |
| Guess 4 | 4 | 6 | $5 = 5$ | $7 \neq 2 \times 3$ |
| Guess 5 | 5 | 7 | $6 = 6$ | $8 = 2 \times 4$ |

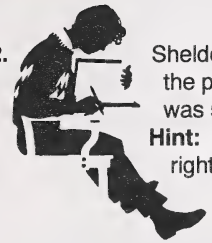
Matt has five cards and Jon has seven cards.

## Practice Activity

### Computer Alternative

1. Do the program "Guess and Test" on Disk A of *Mac 7* (Houghton-Mifflin).

### Print Alternative

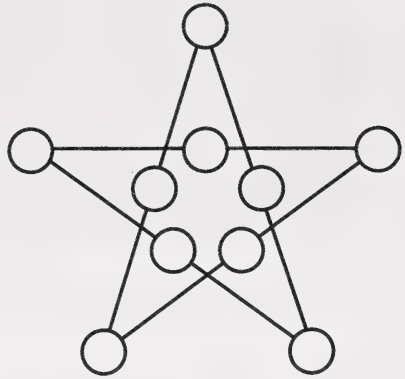


2. Sheldon opened his math texbook and noticed that the product of the numbers of the two open pages was 5852. To what pages was the book opened?
**Hint:** The left page number of a book is even, the right page number is odd.

3. The sum of the ages of a student and teacher is 42. Three years ago the teacher was twice as old as the student. What are their present ages?

4. Barbara bought balloons for the school dance. She spent $6.15 for 23 balloons. If red balloons cost 25¢ each, white balloons cost 15¢ each, and black balloons cost 65¢ each, how many of each kind did she buy?

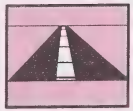
5. Place the numbers 11, 12, 13, 14, 15, 16, 17, 18, 19, and 20 in the circles so that the sum of the numbers in each row is the same.



6. Three bowls have capacities of 11 L, 6 L, and 2 L. The bowls have no markings. How can you use these bowls to measure exactly 7 L of rice?

Turn to the Appendix to check your answers.

# FINDING AND APPLYING A PATTERN

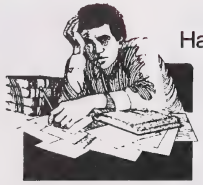


## What Lies Ahead

In this section you will learn these skills.

- making a simpler model of a complex problem in order to find a pattern
- applying a pattern to solve a problem

## Working Together



Have you ever been frustrated because a problem seems too complex? This section deals with one way to simplify a problem: finding a pattern and applying it to the problem.

## Video Activity

Watch Story 1 of *MATH WORKS: Problem Solving: Simplifying the Problem* (Agency for Instructional Television) and Story 2 of *SOLVE IT: Solving a Simpler Problem* (Agency for Instructional Television).

## Program Summary

***MATH WORKS: Problem Solving: Simplifying the Problem* – Story 1**

There are 50 people at a party. Each person wants to shake hands with everyone else. The host points out that it will be a complex problem to find out exactly how many handshakes will be exchanged.

The host demonstrates that the problem can be simplified by using a simpler model. When a smaller number of people is used, a pattern can be discovered.

The problem was acted out with six people and the results were recorded.

| People | Handshakes | Pattern |
|---|---|---|
| 1 | 0 | |
| 2 | 1 | +1 |
| 3 | 3 | +2 |
| 4 | 6 | +3 |
| 5 | 10 | +4 |
| 6 | 15 | +5 |

Now you can apply the pattern. Since $15 + 6 = 21$, seven people will exchange 21 handshakes. Since $21 + 7 = 28$, eight people will exchange 28 handshakes.

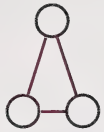
Once you know the pattern, you can calculate how many handshakes will be exchanged by 50 people. Use a calculator.

The answer is 1225 handshakes.

### *SOLVE IT: Solving a Simpler Problem* – Story 2

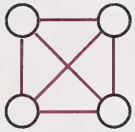
Tree, Sam, and Mike must figure out a bowling schedule for 20 students. Each student must bowl against every other student. They decide to use a simpler model to calculate how many games are needed.

First they consider a schedule for three students. They make a diagram of this. The circles represent the students. The lines represent the games needed.



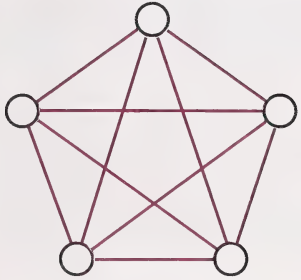
Three games are needed.

Then they consider a schedule for four students.



Six games are needed.

Then they consider a schedule for five students.



Ten games are needed.

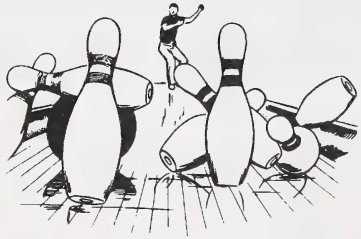
Next they look for a pattern that they can use to solve the problem involving 20 students.

| Number of Students | Games Needed | Pattern |
|---|---|---|
| 1 | 0 | |
| 2 | 1 | +1 |
| 3 | 3 | +2 |
| 4 | 6 | +3 |
| 5 | 10 | +4 |

Next they apply the pattern. Since $10 + 5 = 15$, six students require 15 games. Since $15 + 6 = 21$, seven students require 21 games.

You can use the pattern to find how many games are needed by 20 students. You can use a calculator to solve the problem.

The answer is 190 games.



## Practice Activities

### Computer Alternative

**1.** The computer program you will be using in Question 2 has a built-in calculator to help you do computations. The following exercise will show you how to use this built-in calculator.

You will be working with a computer; so take this booklet and Disk 1 of the *MATH STRATEGIES: Problem Solving* (SRA) with you. Run the disk and wait until you see the chapter menu on the screen.

Type [1]. Press [RETURN]. Follow the directions on the screen until you see the following chart.

| | | |
|---|---|---|
| **Key:** | Plus | + |
| | Minus | – |
| | Times | × or * |
| | Divided by | / |
| | Is equal to | RETURN |

You may type either [×] or [*] for multiplication. Also, since there is no [÷], use [/] for division. You need to use [SHIFT] when typing [+] or [*].

Type [4] [6] [+] [3] [8]. Press [RETURN]. Here's what should be on the screen:

46 + 38 = 84

On the next line, you'll see a colon followed by a flashing dash. This is the signal that the calculator is ready for the next calculation.

Try holding [SHIFT] and pressing [=]. As you can see, the equal sign is not used on this calculator. Whenever you're ready for the calculator to show the answer, press [RETURN] instead.

Type [8] [7] [6]. Then press [←] once. The 6 is erased. When you want to erase something you have typed, use [←].

**Note:** Once you have pressed [RETURN], you cannot erase what you have typed.

To erase all the work that is on the screen, type [E]. Try it.

If you want to type a five-digit number, for example 34 672, you will have to type [3] [4] [6] [7] [2]. The calculator will not accept spaces.

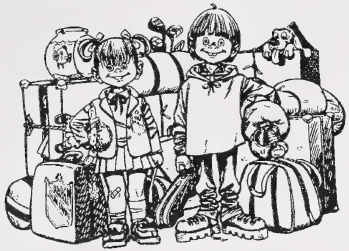
**2.** Choose "5 Using Models" from Disk 1 of the *MATH STRATEGIES: Solving Problems* (SRA) package. Then do the "Spy Ring Problem."

**Note:** When you see a colon followed by a flashing dash on the screen, it means that you can use the built-in calculator to do calculations. After you have done the calculations, you must enter your answer and press [RETURN].

**Print Alternative**

For Questions 3 to 5, you may act out the problems or draw diagrams to help find the patterns. Apply the patterns to solve the problems.

**3.** There are 11 students attending a summer camp. If two students were introduced to each other at a time, how many introductions were made?



**4.** A banquet table can seat four people on each side and one person on each end. What is the maximum number that can be seated if 13 of these tables are put together in one long row?

**5.** There are eight teams in a junior high basketball league. Each team will play each of the other teams twice. What is the total number of games to be played in this league?

Turn to the Appendix to check your answers.

## Did You Know?

### Carl Friedrich Gauss

Carl Friedrich Gauss (1777-1855) was one of the greatest mathematicians of all time. He began to show his talent at an early age. His father was a shop owner who had records to keep. Once, three-year-old Carl was looking over his father's shoulder while he was doing his bookkeeping. The boy said to his father, "That arithmetic is wrong." His father checked his figures and discovered that young Carl really had found a mistake.

In school Carl was always causing trouble because the work was too easy for him and he was bored. Once his teacher decided to keep him quiet for a long time by asking him to add all the whole numbers from 1 to 99. The teacher's plan was spoiled because young Carl gave the answer 4950 immediately and without writing anything down!

Here is the way Carl got the answer so quickly. He thought of the numbers 1 to 99 written forwards and backwards like this:

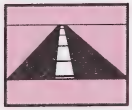
$$1 + 2 + 3 + 4 + \ldots + 96 + 97 + 98 + 99$$
$$99 + 98 + 97 + 96 + \ldots + 4 + 3 + 2 + 1$$

Then he could see that each pair (1 and 99, 2 and 98, and so on) added up to 100. There were 99 such pairs and so the rows together added up to 99 times 100, or 9900. Because Carl wanted the sum of only one row, he divided 9900 by 2 in his head and got the answer 4950.

Carl studied advanced mathematics when he was a teenager and later became a professor at a university in Germany. During his long life he discovered so many new ideas in arithmetic, geometry, and algebra that he has been called a "mathematical giant."[1]

[1] National Council of Teachers of Mathematics for the excerpt from *Mathematical History.*

# SIMPLIFYING A PROBLEM



## What Lies Ahead

In this section you will learn these skills.

- using a similar but simpler problem to help understand a problem
- breaking problems into steps

## Working Together

Are you sometimes confused about which operations to perform in a problem because the numbers in the problem are very large or because the problem seems very complicated? There are strategies you can use to simplify a problem.

## Video Activity

Watch Stories 1 and 3 of *SOLVE IT: Solving a Simpler Problem* (Agency for Instructional Television) and Stories 2 and 3 of *MATH WORKS: Simplifying the Problem* (Agency for Instructional Television) to learn about two simplification strategies.

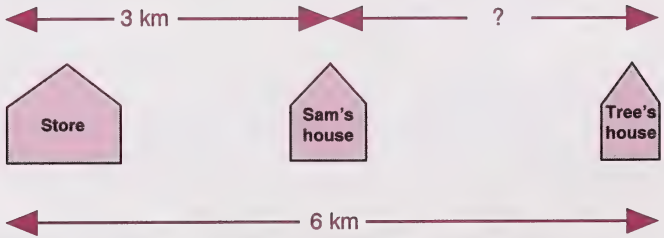
## Program Summary

***SOLVE IT: Solving a Simpler Problem* – Story 1**

Tree, Sam, and Mike want to figure out the average distance from Earth to Mars. They know that the average distance from the sun to Mars is 228 000 000 km and the average distance from the sun to Earth is 150 000 000 km. They are overwhelmed by the large numbers and cannot figure out the problem, so they decide to make up a similar but simpler problem.

**Simpler Problem**

Sam and Tree both live east of the store. Sam lives 3 km from the store and Tree lives 6 km from the store. How far is it from Sam's house to Tree's house?

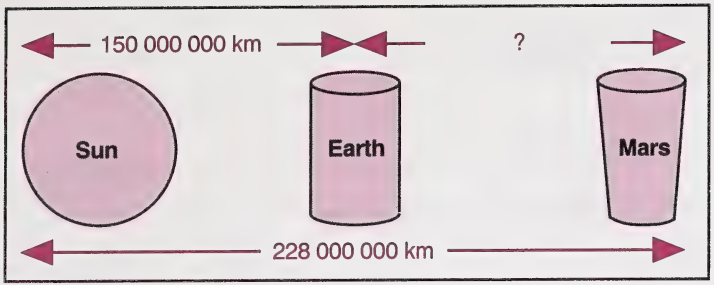



The boys know that the distance between Sam's house and Tree's house is found by subtracting.

$6 - 3 = 3$

It is 3 km from Sam's house to Tree's house.

The simpler problem gives Tree, Sam, and Mike an idea for solving the original problem. They use objects to represent the sun, Earth, and Mars.




Now they understand that the distance between Earth and Mars can be found by subtraction.

228 000 000 – 150 000 000 = 78 000 000

The distance from Earth to Mars is 78 000 000 km.

**Note:** This strategy of making up a simpler but similar model is similar to the strategy you learned in Section 1: changing the setting of a problem to help understand the problem.

### *SOLVE IT: Solving a Simpler Problem* – Story 3

Tree, Sam, and Mike find a coded note and they try to figure out its meaning. They are confused because a number of steps are involved. They decide to decode one piece of the note at a time. By doing this, they succeed in understanding the note.

### *MATH WORKS: Simplifying the Problem* – Story 2

Carlos wants to write a report about all the insects that live in his state. When Carlos discovers that there are 88 600 insect species in North America, he thinks the problem is too big to handle.

The librarian suggests that Carlos should break down the problem into smaller pieces and tackle the pieces step by step.

### *MATH WORKS: Simplifying the Problem* – Story 3

Carlos, Sonia, and Won-Li have volunteered to help the gymnastics coach raise money for a trampoline.

They plan to sell candy, but first they need to find out how much to charge per box. They find the problem difficult until they substitute smaller numbers. By solving a problem with smaller numbers, they discover how to solve the original problem.

## Practice Activity

For Questions 1 to 3, use simpler problems to help you solve the original problems. You can change the settings and the numbers, but do not change the operations.

1. In 1951, $309 465 000 was paid for family allowance in Canada. In 1987, $2 534 420 000 was paid for family allowance. How much did the family allowance payments increase during this time period?

2. In 1986, the total value of Canadian imports was $112 974 000 000. Of this, $18 114 000 000 was spent on engines for motor vehicles. What percent of the total imports was spent on engines? Round your answer to the nearest whole percent.

3. **a.** Canada's land area is 9 976 000 $km^2$ and the land area of the former Union of Soviet Socialist Republic was 22 402 000 $km^2$. How many times larger was the USSR than Canada? Round your answer to the nearest tenth.

   **b.** The Republic of Russia's land area is 17 075 400 $km^2$. How many times larger is Russia than Canada? Round your answer to the nearest tenth.

**Computer Alternative**

4. Do "Lightning Strikes" on Disk 1 from the *MATH STRATEGIES: Solving Problems* (SRA) package.

   When the chapter menu comes on the screen, type 2 and press RETURN. You will then see the menu for "Simplifying Problems."

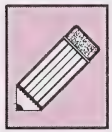
   Type 1 for "Lightning Strikes" and press RETURN.

   Follow the instructions on the screen.

5. Do "A Beautiful Dream" from Disk 2 of *MATH STRATEGIES: Solving Problems* (SRA).

   From the chapter menu, choose "3 Problem Breakdown." Then choose "2 A Beautiful Dream."

   Solve the problem at the computer.



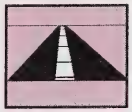
**Print Alternative**

Simplify the following problems by doing them in steps.

6. The students in a Grade 8 class have a total of 51 pets. Five students have one pet each, eight students have three pets each, three students have no pets, and the rest have two pets each. How many students have exactly two pets?

7. Anton charges $10 per hour to clear the neighbours' driveways. His father charges him $2 per hour for use of the snowblower. How much profit does Anton make if he and a friend work 27 h and they share the profits equally?

Turn to the Appendix to check your answers.



## What Lies Ahead

In this section you will learn these skills.

- working backwards to solve problems
- using flow charts and inverse flow charts to solve a problem

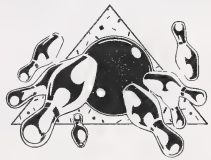
## Working Together

In mathematics you are usually given information, and then asked to find the answer. Sometimes, however, you are given the answer and then you are asked to find a piece of information. In cases like these, you need to work backwards.

Drawing a flow chart and an inverse flow chart is often helpful. A **flow chart** is a diagram that shows a sequence of actions. An **inverse flow chart** shows the backwards sequence of actions.

### Example

Ross plays in a bowling league. His average in the league is 198. Today Ross scored 195 and 175 in his first two games. What minimum score must he make in his third game to maintain his average?



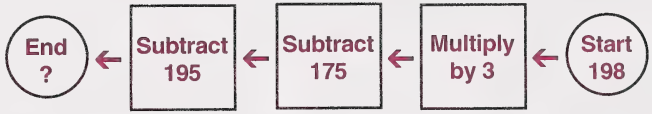
### Solution

First consider how the problem would be done if you worked forwards. This flow chart shows the sequence of the steps.

You start with the score of the third game, add each of the scores of the other two games, divide by 3, and end with the average of 198.

You do not know one of the three scores, so you cannot do the first step, but you can work backwards.

This inverse flow chart shows the backwards sequence of operations. These operations undo operations in the original flow chart.




You start with the average of 198, multiply by 3, subtract 175, and subtract 195. You will end with the minimum score that Ross must score in the third game.

Ross must get at least 224 in the last game.

**Note:** You can check to see if 224 is the correct third score by working forwards.

## Practice Activity 1

1. Bonita works as a controller at a parkade. During the morning she counted 61 cars entering the parkade and 29 cars leaving. During the afternoon she counted 17 cars entering and 54 leaving. If there were 18 cars in the parkade when Bonita left, how many cars were in the parkade when she started?

2. On Monday morning, a store put out some watermelons, and six were sold. On Tuesday morning, the number of leftover watermelons was doubled, and six were sold. On Wednesday morning, the number was tripled, and six were sold. If there were no watermelons left on Wednesday afternoon, how many watermelons were put out on Monday morning?

3. Jeanne went shopping. She spent $54 in a clothing store. Then she stopped by her mother's office. Her mother gave her $20. Jeanne then spent $36 in a shoe shop. She returned home with $8. How much money did she have when she first went shopping?

4. Pierre has been spending about 6 h and 20 min each week practising the piano. On Monday he practised for 1 h and 10 min, on Tuesday he practised 1.5 h, on Wednesday he practised 1 h and 50 min, and on Thursday he practised 1 h. How much time must he spend practising the piano on the weekend to maintain his average?

Turn to the Appendix to check your answers.

## Did You Know?

### Aryabhata

The ancient Hindus loved to do number puzzles. A mathematician name Aryabhata, who lived in India during the sixth century A.D., liked this kind of puzzle: If 4 is added to a certain number, the result divided by 2, that result multiplied by 5, and then 6 subtracted from that result, the answer is 29. Can you find the number?

If you did find the number, maybe you did it just by trying out certain numbers until you found one that worked. Aryabhata had another way to find the number. It is called the method of inversion. This means he worked backwards and did the inverse, or opposite, steps as he went along. Adding and subtracting are inverse steps, and multiplying and dividing are inverse steps.

Look back at the puzzle. Aryabhata started with 29 and added 6 to get 35. Then he divided by 5 to get 7. Then he multiplied by 2 to get 14. Then he subtracted 4 to get 10, which is the number. You can check to see that 10 is the number you want.

Use Aryabhata's method in each of the following problems. The answers are in the Appendix.

1. If a number is divided by 6, the result multiplied by 5, and 8 added to that result, the answer is 23. Find the number.

2. If 10 is added to a number, then 3 added to the result, that result divided by 4, that result multiplied by 7, and 2 subtracted from that result, the answer is 40. Find the number.

3. If a number is multiplied by 2, 10 added to the result, 8 subtracted from that result, and that result multiplied by 3, the answer is 6. Find the number. [1]

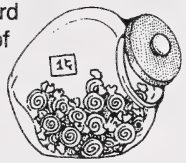
[1] National Council of Teachers of Mathematics for the excerpt from *Mathematical History.*

## Working Together

The following examples also use the strategy of working backwards. However, they are a little more complicated.

### Example 1

Ruth has a jar of candies. She gives Rachel one-third of her candies. She then gives Hannah one-fourth of the remaining candies. Finally she gives one-fifth of the candies that she has left to Janice. If Ruth has 12 candies left at the end, how many did she have at the beginning?



### Solution

The order of operations can be shown in a flow chart if you are aware of the following relationships.

- Giving away one-third of the candies is the same as multiplying the number of candies by two-thirds.
- Giving away one-fourth of the remaining candies is the same as multiplying by three-fourths.
- Giving away one-fifth of the remaining candies is the same as multiplying by four-fifths.

If you are unaware of this relationship, you should act out similar problems to this.

> Ruth has 12 candies. She gives Rachel one-third of her candies. How many candies does she have left?

**Method 1**

$\frac{1}{3} \times 12 = 4$

$12 - 4 = 8$

Ruth has eight candies left.

**Method 2**

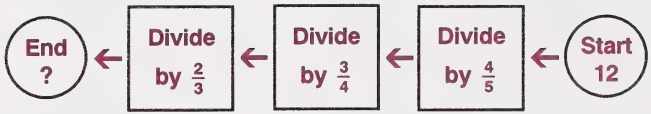
$1 - \frac{1}{3} = \frac{2}{3}$

$\frac{2}{3} \times 12 = 8$

Ruth has eight candies left.

To solve the original problem, make a flow chart.

Now work backwards. An inverse flow chart will be helpful.




Ruth had 30 candies at the beginning.

## Example 2

Curly gave Larry and Moe as much money as each had. Then Larry gave Curly and Moe as much money as each had. Then Moe gave Curly and Larry as much money as each had. If each one now has $8, how much did each have at the beginning?

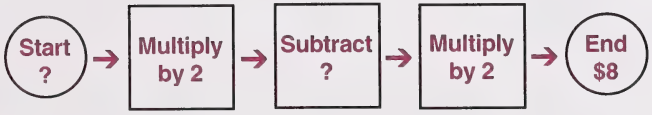
## Solution

This problem is more complicated because three people are involved and three operations are occurring at each step. Three flow charts are required.

Be careful. Giving people as much as they have doubles their money.

**Curly**

**Larry**




**Moe**

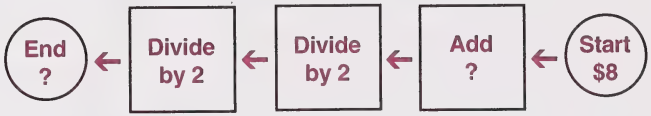
Notice that the subtracted amount is unknown in each step.

To solve the problem, work backwards. An inverse flow chart will be helpful.

**Curly**

**Larry**

**Moe**




Follow the steps in the inverse flow chart. You may wish to act this problem out using paper money.

**Step 1**

Curly, Larry, and Moe each have $8.

Curly and Larry's money is divided by 2. They each have $4 now $(8 \div 2 = 4)$.

The amount which Curly and Larry lost is given to Moe $(4 + 4 = 8)$.

Replace the question mark in the box in Moe's inverse flow chart with 8.

Moe has \$16 now $(8+8=16)$.

**Step 2**

Curly and Moe's money is divided by 2. Curly has \$2 now $(4\div 2=2)$. Moe has \$8 now $(16\div 2=8)$.

The amount Curly and Moe lost is given to Larry $(2+8=10)$.

Replace the question mark in the box in Larry's inverse flow chart with 10.

Larry has \$14 now $(4+10=14)$.

**Step 3**

Larry and Moe's money is divided by 2. Larry has \$7 now $(14\div 2=7)$. Moe has \$4 now $(8\div 2=4)$.

The amount Larry and Moe lost is given to Curly $(7+4=11)$.

Replace the question mark in the box in Curly's inverse flow chart with 11.

Curly has \$13 now $(2+11=13)$.

**Summary**

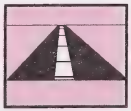
So, at the beginning Curly had \$13, Larry had \$7, and Moe had \$4.

## Practice Activity 2

1. The local shelter did not receive any new animals last week, but they gave a lot of animals away. On Monday they gave away one-third of the pets in the shelter. On Tuesday they gave away nine pets. On Wednesday they gave away one-third of the remaining pets. On Thursday they gave away nine pets. On Friday only nine pets remained. How many pets were there at the beginning of the week?

2. Sara gave Barb and Shae half as much money as each had. Barb then gave Sara twice as much as she had, and Barb gave Shae \$1. Next Shae gave Barb \$1 and Sara \$3. If each one now has \$12, how much did each have at the beginning?

Turn to the Appendix to check your answers.



## What Lies Ahead

In this section you will learn these skills.

- using a table to organize clues
- using logical reasoning to eliminate clues and solve a problem

## Working Together

Detectives are skillful at using logical reasoning to eliminate clues and solve mysteries. You can become skillful at using logical reasoning to solve problems too.

## Video Activity

Watch *SOLVE IT: Using Logical Reasoning* and then read the notes in this section.

## Program Summary

### Story 1

Rita and Marcus have four gift-wrapped presents and four cards. They are unsure which card goes with which gift. They remember three clues about the gifts: Grandpa's gift is cologne, Grandma's gift is perfume and it is not wrapped in blue paper, and Baby Christopher's gift is a stuffed walrus.

Rita and Marcus use a careful process of elimination to discover which gift belongs to Grandpa, Grandma, Baby Chris, and Aunt Roxy.

First they make a table of the gifts and who could possibly receive them.

| Gifts | Grandpa | Grandma | Baby Chris | Aunt Roxy |
|---|---|---|---|---|
| 1. Large blue box that does not swish | | | | |
| 2. Small blue box that swishes | | | | |
| 3. Small box that swishes. It is not blue. | | | | |
| 4. Small box that does not swish. It is not blue. | | | | |

Then Rita and Marcus decide which gifts cannot belong to each person. For example, gifts 2, 3, and 4 are too small to be Baby Chris' stuffed walrus.

Gifts 1 and 4 do not swish, so they cannot be Grandpa's cologne or Grandma's perfume. Gift 2 cannot be Grandma's because it is wrapped in blue paper.

| Gifts | Grandpa | Grandma | Baby Chris | Aunt Roxy |
|---|---|---|---|---|
| 1. Large blue box that does not swish | X | X | | |
| 2. Small blue box that swishes | | X | X | |
| 3. Small box that swishes. It is not blue. | | | X | |
| 4. Small box that does not swish. It is not blue. | X | X | X | |

Marcus and Rita discover the correct combinations through elimination. Gift 1 must belong to Baby Chris and gift 3 must belong to Grandma. Since Grandma's gift is now known, the other small package that swishes must be Grandpa's. That leaves the final gift for Aunt Roxy.

**Story 2**

Lesley and Mike are selling tie racks for $4, stools for $5, and shoeshine boxes for $6.

Lesley makes the sales and records the number of each item that has been ordered in a chart like this.

| Tie Racks | Stools | Shoeshine Boxes |
|---|---|---|
| IIII | 𝍸 I | III |

She also records the name of each customer and the amount of money they have paid in advance.

| Name | Payment ($) |
|---|---|
| Mrs. Kwon | 18 |
| Mrs. Emry | 10 |
| Ms. Grenvier | 13 |
| Mr. Gresure | 9 |
| Mrs. Chandler | 14 |

Unfortunately, Lesley did not write down what each person ordered, so Mike cannot make the deliveries. However, they are able to solve the problem through a process of elimination. They make a chart like this.

| Money Paid ($) | Number of Items | | |
|---|---|---|---|
| | Tie Racks ($4) | Stools ($5) | Shoeshine Boxes ($6) |
| 9 | | | |
| 10 | | | |
| 13 | | | |
| 14 | | | |
| 18 | | | |
| | 4 | 6 | 3 |

Lesley and Mike realize that there is only one way that an order of $9 can be filled – with a tie rack and a stool $(\$4+\$5)$. There is also only one way an order of $13 can be filled – with two tie racks and one stool $(\$4+\$4+\$5)$. The other orders can be filled in more than one way.

- The $10 order could have been for two stools $(\$5+\$5)$, or for a tie rack and a shoeshine box $(\$4+\$6)$.
- The $14 order could have been for two tie racks and a shoeshine box $(\$4+\$4+\$6)$ or for two stools and a tie rack $(\$5+\$5+\$4)$.
- The $18 order could be for three shoeshine boxes $(\$6+\$6+\$6)$, for two tie racks and two stools $(\$4+\$4+\$5+\$5)$, or for three tie racks and a shoeshine box $(\$4+\$4+\$4+\$6)$.

Then Lesley remembers that Mr. Kwon bought three shoeshine boxes.

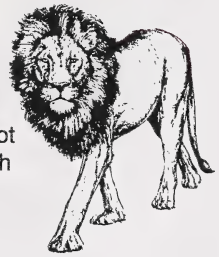
Lesley and Mike record what is known in the table. They list the other possibilities at the right of the table.

| Money Paid ($) | Number of Items | | | |
|---|---|---|---|---|
| | Tie Racks ($4) | Stools ($5) | Shoeshine Boxes ($6) | |
| 9 | 1 | 1 | | |
| 10 | | | | $\$5+\$5$ or $\$4+\$6$ |
| 13 | 2 | 1 | | |
| 14 | | | | $\$4+\$4+\$6$ or $\$5+\$5+\$4$ |
| 18 | | | 3 | |
| | 4 | 6 | 3 | |

Lesley realizes that the $10 order and the $14 order must be one tie rack and four stools since all the other items are accounted for. Through elimination, they figure out that the $10 order was for two stools, and the $14 order was for two stools and one tie rack.

## Practice Activity

1. Some animals had a race over a short distance. Neither the grizzly bear nor the coyote beat the lion. The coyote beat the rabbit, but not the ostrich. The grizzly bear lagged behind the rabbit. If the ostrich was not the fastest, in what order did the animals finish the race?



2. Pedro, Patti, and Penelope each ate a pair of pears. The six different varieties of pears consumed were Anjou, Bosc, Bartlett, Clapp's Favorite, Comice, and Seckel. Only one person ate two pears whose varieties started with the same letter, and that person was one of the girls. Pedro ate the Bartlett, but not the Seckel. Penelope ate the Comice pear. Who ate which pair of pears?

Turn to the Appendix to check your answers.

## Did You Know?

### Mary Fairfax Somerville

Mary Fairfax Somerville almost lost out on learning mathematics because she was a girl. She was born in Scotland in 1780 during a time when girls there were supposed to learn only such things as cooking and sewing. She did cook and sew, but she was much happier when she could run and play on the beach near her home.

Her parents tried to make her more of a lady by sending her to a girls' school. There she had to wear a tight corset, learn how to paint, and memorize a whole page in a dictionary. She hated this school.

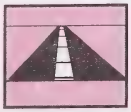
Once in a magazine she saw an arithmetic problem. She got so excited about it that she wanted to learn mathematics – something girls didn't do! She happened to hear someone talking about Euclid, one of the great Greek mathematicians, and she decided to try to buy a copy of his book. Even this was hard for her to do because girls did not go into stores. Her brother's teacher got the book for her. She studied it very hard, and a whole new world of ideas opened up to her. One day she helped her brother with a problem, much to the surprise of his teacher.

After this, Mary studied many other things on her own. By the time she married her cousin, William Somerville, in 1812, she had learned about calculus and astronomy. Calculus is a hard part of mathematics, and astronomy is the study of the way the stars move. Dr. Somerville introduced his wife to many scientists and mathematicians who were happy to talk to Mary about her studies.

She wrote several books about mathematics and science. She also found time to raise a family. Mary Fairfax Somerville was still studying mathematics on the day she died at the age of ninety-two.[1]

[1] National Council of Teachers of Mathematics for the excerpt from *Mathematics History.*

# USING VENN DIAGRAMS



## What Lies Ahead

In this section you will learn these skills.

- using logical reasoning and Venn diagrams to group information and solve problems

## Working Together

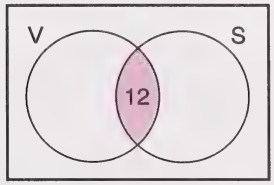
In the previous sections you have used diagrams to organize information. A type of diagram that is helpful in some problems is a **Venn diagram**. In this special kind of diagram, circles are used to represent groups of people, animals, or objects that have certain characteristics. The positioning of the circles in relation to one another represents relationships among these groups. These diagrams were named after an English mathematician named John Venn (1834-1923).

### Example 1

There are 28 girls on the volleyball team and 19 girls on the swim team. If 12 girls belong to both teams, how many girls belong to the swim team only?

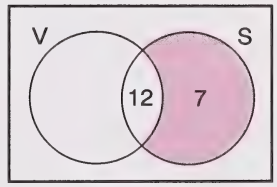
### Solution

Represent the members of the volleyball team with a circle. Represent the members of the swim team with another circle. Represent the members of both teams by the intersection of these two circles.




You can now calculate the number of girls who only belong to the swim team.

$19 - 12 = 7$



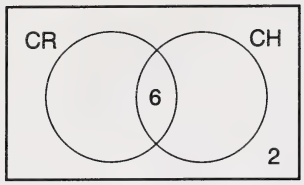

So, seven girls belong to the swim team only.

### Example 2

Reilly bought some plates at a yard sale. He discovered that 10 plates were cracked, 12 plates were chipped, 6 plates were both chipped and cracked, and 2 plates were neither chipped nor cracked. How many plates did Reilly buy?

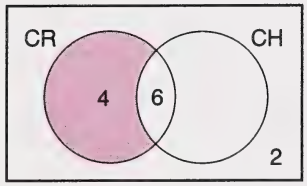
**Solution**

Represent the cracked plates with a circle. Represent the chipped plates with a second circle. Represent the cracked and chipped plates with the intersection of the two circles. Represent the undamaged plates with the region outside the circles.



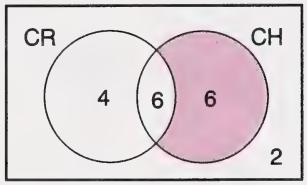

Now calculate the number of plates that were only cracked.

$10 - 6 = 4$



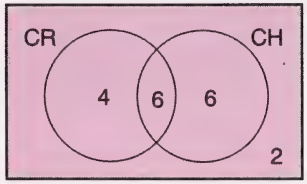

Now calculate the number of plates that were only chipped.

$12 - 6 = 6$



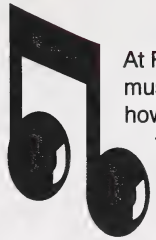

Now calculate the total number of plates that were bought.

$4 + 6 + 6 + 2 = 18$




So, 18 plates were bought.

## Practice Activity 1



1. At Parkdale Junior High, students must enroll in either music or art, or they can take both. In a class of 32, how many students are enrolled in art if 11 students take only music and 9 students take both art and music?

2. In a group of students, 25 students brought an apple in their lunch, 20 students brought a cookie in their lunch, 14 students brought both an apple and a cookie, and 3 students brought neither an apple nor a cookie. How many students were there altogether?

Turn to the Appendix to check your answers.

## Working Together

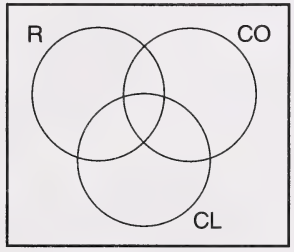
The following example uses Venn diagrams with three circles to solve a problem.

### Example

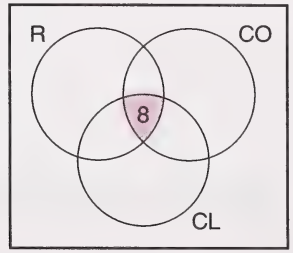
In a survey of 100 people, the following information was collected: 43 people like country music, 55 like rock music, 45 like classical music, 8 people like all three forms, 15 people like both country music and classical music, 25 like both rock and country music, and 20 like both rock and classical music. How many people do not like any of these forms?

### Solution

Use three circles to show the people who like country music, rock music, and classical music.



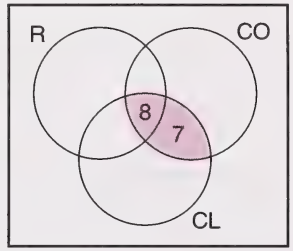

The intersection of the three circles shows the people who like all three forms. The survey states that 8 people like all three forms.




The intersection of the country and classical circles shows the people who like both country and classical music.

Calculate the number of people who like country and classical music, but not rock music.

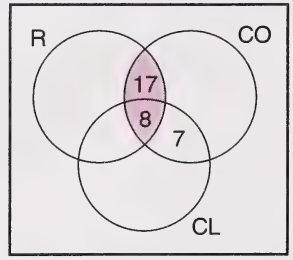
$15 - 8 = 7$




The intersection of the country and rock circles shows the number of people who like both country and rock music.

Calculate the number of people who like both country and rock music, but not classical music.

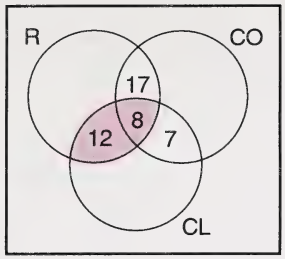
$25 - 8 = 17$



The intersection of the rock and classical circles shows the number of people who like both rock and classical music.

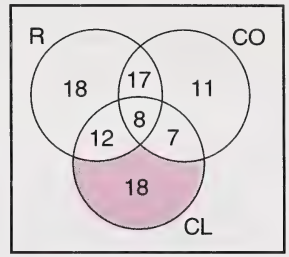
Calculate the number of people who like both rock and classical music, but not country music.

$$20 - 8 = 12$$



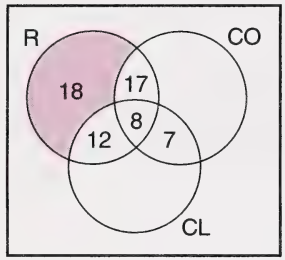

Now that you know the number of people who like classical music and another form of music, calculate the number of people who like only classical music.

$$45 - (12 + 8 + 7) = 18$$



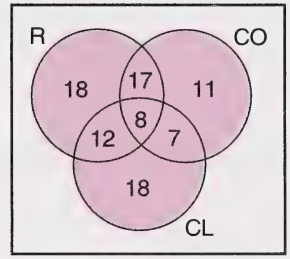

Now that you know the number of people who like rock music and another form of music, calculate the number of people who only like rock.

$$55 - (17 + 8 + 12) = 18$$



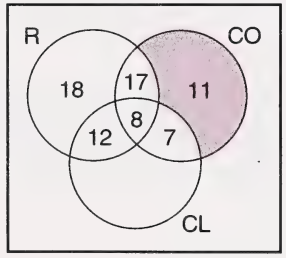

Now you can calculate the total number of people who like rock, country, or classical music.

$$18 + 17 + 8 + 12 + 11 + 7 + 18 = 91$$



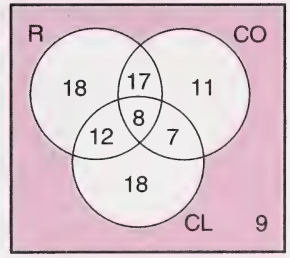

Now that you know the number of people who like country music and another form of music, calculate the number of people who only like country music.

$$43 - (17 + 8 + 7) = 11$$




Since 100 people were surveyed, you can find the number of people who do not like any of these forms of music.

$$100 - 91 = 9$$

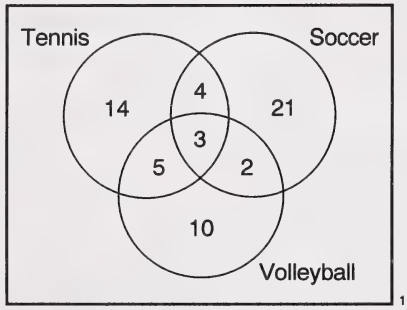



Of the 100 people surveyed, 9 people do not like any of these forms of music.

## Practice Activity 2

1. A survey was taken to find how many students play certain sports. According to the Venn diagram, how many students play each of the following sports?

   **a.** tennis
   **b.** tennis but not soccer or vollyball
   **c.** soccer
   **d.** soccer but not tennis or volleyball
   **e.** volleyball
   **f.** volleyball but not tennis or soccer
   **g.** both tennis and soccer
   **h.** both tennis and volleyball
   **i.** both soccer and volleyball
   **j.** all three sports




[1]

2. In a neighbourhood of 50 families, the following facts are known: 25 families have cats, 27 have dogs, 18 have tropical fish, 5 have only cats and dogs, 3 have only dogs and tropical fish, 2 have only cats and tropical fish, and 8 have all three kinds of pets. How many of the families have none of these kinds of pets?

Turn to the Appendix to check your answers.

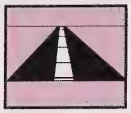
## Did You Know?

### Charles Dodgson

Reverend Charles Dodgson (1832-1898), a well-known mathematician and logician at Oxford University in England, was also famous for his writing ability. He wrote *Alice's Adventures in Wonderland* and *Through the Looking Glass* using the pen name Lewis Carroll.[2]

[1] 1989 Creative Publication's, Sunnyvale, California 94086 for excerpts from *Middle School with Pizzazz.*

[2] National Council of Teachers of Mathematics for the excerpt from *Mathematics History.*



## What Lies Ahead

In this section you will learn these things.

- There are many ways to solve one problem.
- There may be more than one solution.

## Working Together

In this module you have been learning strategies for solving problems. You have probably discovered that there are many ways to solve a particular problem. This section stresses this idea.

### Example 1

There are ten animals in a barnyard. Some are chickens and the rest are sheep. Ruth counts 28 legs. How many chickens and how many sheep are there? **Hint:** Chickens have two legs and sheep have four legs.

### Solution

### Method 1

You could guess, check, and revise.

| Animals | Guess 1 | Guess 2 | Guess 3 |
|---|---|---|---|
| Chickens | 2 | 4 | 6 |
| Sheep | 8 | 6 | 4 |
| Legs | 36 | 32 | 28 |

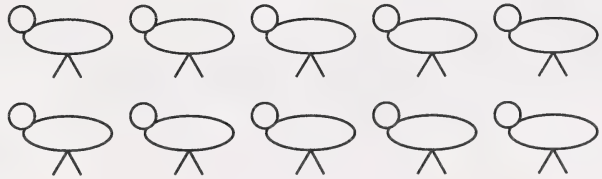
↑
Guess 3 satisfies both conditions.

There are six chickens and four sheep.

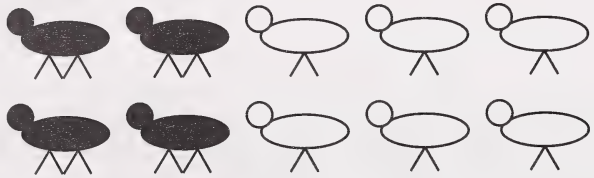
### Method 2

You could make a diagram.

There are ten animals and every animal has at least two legs.



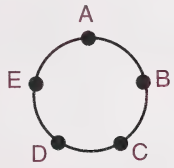
This group of animals had 28 legs, so you need to add eight legs to the drawing. Add the legs in pairs to each animal picture.



There are four sheep and six chickens.

### Example 2

How many chords can be drawn joining five points on a circle?
**Hint:** A chord is a line segment that joins two points on the circumference of a circle.




### Solution

### Method 1

You could use simpler models and look for a pattern.

| Number of Points | Number of Chords | Pattern |
|---|---|---|
| 1 | 0 | |
| 2 | 1 | +1 |
| 3 | 3 | +2 |
| 4 | 6 | +3 |

You can apply this pattern. A circle with five points has ten chords.

### Method 2

Make an organized list of the chords from each point.

The chords from A: AB, AC, AD, AE

The chords from B: BC, BD, BE, ~~BA~~ (This has already been mentioned.)

The chords from C: CD, CE, ~~CA~~, ~~CB~~ (This has already been mentioned.)

The chords from D: DE, ~~DA~~, ~~DB~~, ~~DC~~ (This has already been mentioned.)

The chords from E: ~~EA~~, ~~EB~~, ~~EC~~, ~~ED~~ (This has already been mentioned.)

Count the chords in the organized list.

There are ten chords joining five points.

## Practice Activity 1

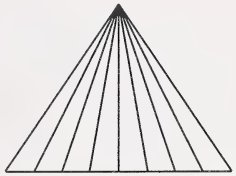
### Computer Alternative

1. Do the programs "Diagonals" and "Squares" in the *MECC: Problem Solving Strategies* package.

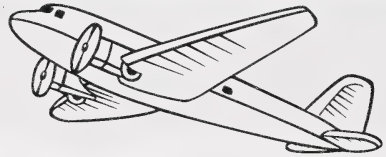
### Print Alternative

For Questions 2 and 3, use two different methods to solve the problem.

2. How many triangles are in this figure?



3. Meredith watched 27 planes land at the airport. She noticed that some planes had two engines and some had four. If she counted 70 engines, how many four-engine planes did she see?



Turn to the Appendix to check your answers.

## Working Together

Sometimes there is more than one solution to a problem.

**Example**

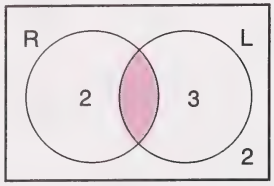
In a certain cave there are seven bats. Of these bats, two bats could see out of the right eye, three could see out of the left eye four could not see out of the left eye, and five could not see out of the right eye. How many bats in the cave could see out of both eyes?

**Solution**

You can use Venn diagrams to organize the information. There are three possible solutions.

### Solution 1

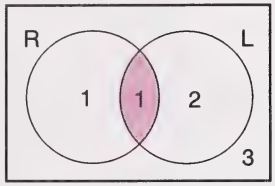
There are no bats that can see out of both eyes.




There are two blind bats in the cave.

### Solution 2

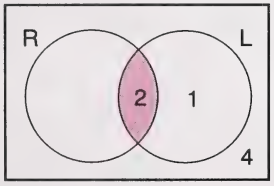
There is one bat that can see out of both eyes.




There are three blind bats in the cave.

### Solution 3

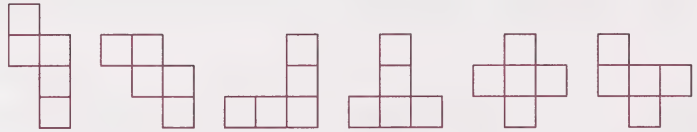
There are two bats that can see out of both eyes.

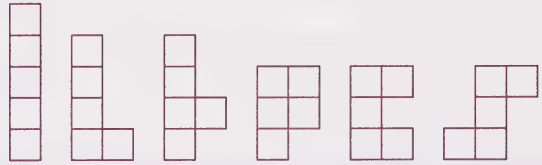



There are four blind bats in the cave.

## Practice Activity 2

A **pentomino** is formed by five congruent squares that share a common side. There are twelve different types of pentominos.





Glue a sheet of graph paper to cardboard. Then draw the twelve pentominos on the graph paper and cut them out. If you use the pentaminos like jigsaw puzzle pieces, how can you assemble them into a rectangle? **Hint:** There is more than one solution.

Turn to the Appendix to check your answers.

# MODULE CONCLUSION

## What Lies Ahead

In the module conclusion you will review the module and do the module assignment.

## Working Together

In Module 1 you learned the four stages of problem solving and these skills.

- identifying the essential elements of a problem (what you know and what you need to know)
- ignoring unneccessary details
- changing the setting of a problem to help understand the problem
- restating the problem in your own words
- estimating an answer
- determining if an answer is reasonable
- using sketches to organize information
- using objects to act out a problem or organize information
- making organized lists to solve problems
- using tree diagrams to make an organized list
- making tables to organize information
- solving problems by guessing, checking, and revising your answer
- organizing your guesses in a table
- making a simpler model of a complex problem in order to find a pattern
- applying a pattern to solve a problem
- using a similar but simpler problem to help you understand a problem
- breaking the problem into steps
- working backwards to solve problems
- using flow charts and inverse flow charts to solve a problem
- using a table to organize clues
- using logical reasoning to eliminate clues and solve a problem
- using logical reasoning and Venn diagrams to group information and solve problems

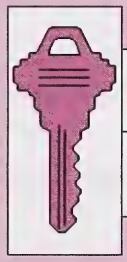
Now that you have studied Module 1, you should be ready to do the assignment for Module 1.

## Module Assignment

Turn to the Assignment Booklet and complete the Module Assignment. You may use your notes, but do the assignment independently.

Afterwards, submit the assignment for a grade and feedback from your learning facilitator.

# APPENDIX



**Glossary**

**Suggested Answers**

## Glossary

**Flow chart:** a diagram that shows a sequence of events or instructions

**Inverse flow chart:** a diagram that shows a backwards sequence of events or instructions

**Problem:** a task for which the method for finding the answer (as well as the answer) is not immediately known

**Tree diagram:** a branching diagram used to make an organized list

**Venn diagram:** a special kind of diagram in which circles are used to represent groups of people, animals, or objects that have certain characteristics

## Suggested Answers

### Section 1: Practice Activity

1. **a.** Marcel is working ~~very hard~~ to earn enough money to buy a new $425 bicycle. He gets paid $14 per week for babysitting ~~two hours on Thursday and five hours on Saturday~~. As well, he earns $20 per week for delivering newspapers ~~between 6:30 and 7:30 a.m. every morning~~. He also gets paid $5 ~~every Wednesday~~ for ~~spending 1.5 h~~ delivering ~~weekly advertising~~ flyers. For how many weeks will Marcel have to work so that he can buy his new bicycle?

   Marcel wants to buy a bicycle that costs $425. He earns $14 per week for babysitting, $20 per week for delivering papers, and $5 per week for delivering flyers. For how many weeks will Marcel have to work before he can buy his bike?

   **b.** A plane ~~leaves Vancouver at 7:30 a.m. and~~ arrives in Toronto at 12:00 noon Vancouver time. ~~It flies a distance of 3400 km at a cruising speed of 950 km/h.~~ When it is 6:00 a.m. in Vancouver, it is 9:00 a.m. in Toronto. What time is it in Toronto when the plane lands?

   A plane arrives in Toronto at 12:00 noon Vancouver time. When it is 6:00 a.m. in Vancouver, it is 9:00 a.m. in Toronto. What time is it in Toronto when the plane lands?

2. **a.** Nina bought a sweater for $30 and then sold it to a friend for $40. She bought the sweater back from her friend for $60 and sold it to another friend for $70. How much money did Nina make or lose while buying and selling the sweater?

   **b.** Thor's shirt cost twice as much as Marc's shirt. If Marc's shirt cost $30, how much did Thor's shirt cost?

### Section 2: Practice Activity

1. If you assume that the person will take two steps to cover 1 m, 2000 steps will be required to cover 1000 m, which is equivalent to 1 km.

   The answer is **C**.

2. If 25 people could consume 50 patties, and each patty weighs 100 g, the total consumed will be 5000 g, which is equal to 5 kg.

   The answer is **A**.

3. If you use 6 mL of toothpaste each day, you will use 42 mL of toothpaste each week. 100 mL of toothpaste should last about two weeks.

   The answer is **A**.

**4.** $29 \div 4 = 7\,R1$

If seven cars are used, one student will be left behind. Since all 29 students must go, eight cars must be used.

The answer is **D**.

**5.** $29 \div 4 = 7\,R1$

Only one tape is in the last row since 28 tapes are required to complete seven rows.

The answer is **B**.

**6.** $29 \div 4 = 7.25$

An exact answer is needed here. Fractions of an hour are possible.

The answer is **C**.

**7.** $29 \div 4 \doteq 7$

An estimate is required here.

The answer is **A**.

**8.** **Estimate**

$$95.82 + 53.21 + 9.58 + 64.49 \doteq 100 + 50 + 10 + 60$$
$$\doteq 220$$

The cost is more than \$200. So, Shamaz will not be able to buy all these items.

**Calculation**

$95.82 + 53.21 + 9.58 + 64.49 = 223.10$

The cost is \$223.10. So, Shamaz will not be able to buy all the items.

**9.** **Estimate**

$182 \times 18 \doteq 200 \times 20$
$\doteq 4000$

About 4000 tickets will be sold.

**Calculation**

$182 \times 18 = 3276$

Exactly 3276 tickets will be sold.

**10.** **Estimate**

$350 \div 0.25 \doteq 400 \div \frac{1}{4}$
$\doteq 400 \times 4$
$\doteq 1600$

About 1600 buttons were sold.

**Calculation**

$350 \div 0.25 = 1400$

Exactly 1400 buttons were sold.

**11.** **Estimate**

$9895 \div 12 \doteq 9600 \div 12$
$\doteq 800$

The payments excluding interest will be about \$800 each month for one year.

**Calculation**

$9895 \div 12 = 824.59$

The payments will be \$824.59 each month for one year.

## Section 3: Practice Activity

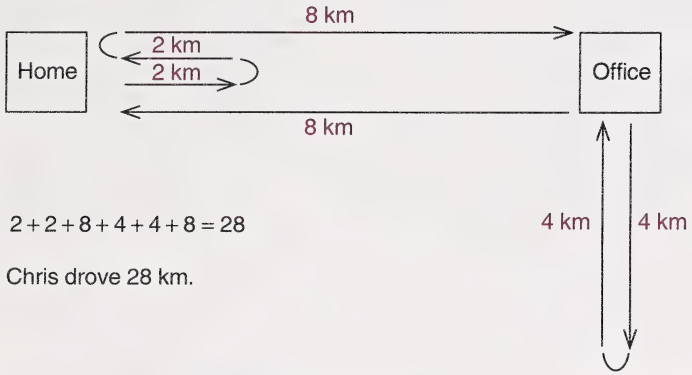
### Computer Alternative

**1.** computer checked

**2.** computer checked

### Print Alternative

**3.**




$2+2+8+4+4+8=28$

Chris drove 28 km.

**4.** Use objects to act out the problem.

**Trip 1:** The farmer leaves the fox and corn and he takes the duck across the river.

**Trip 2:** He leaves the duck on the other side of the river and returns across the river.

**Trip 3:** The farmer leaves the corn, picks up the fox, and goes across the river with the fox.

**Trip 4:** He leaves the fox on the other side, picks up the duck, and returns across the river with the duck.

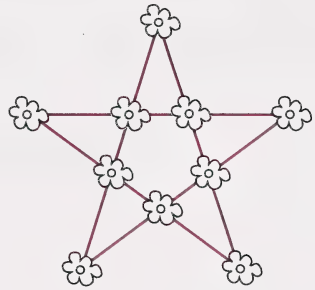
**Trip 5:** The farmer leaves the duck, picks up the corn, and goes across the river with the corn.

**Trip 6:** He leaves the corn with the fox and returns across the river.

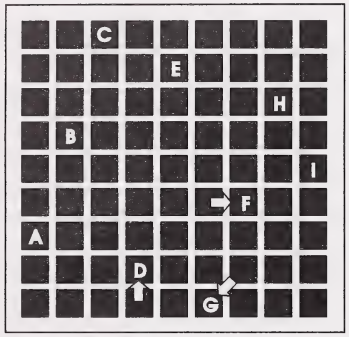
**Trip 7:** The farmer picks up the duck and takes the duck across the river.

Finally, all the animals and the corn are on the other side of the river with the farmer.

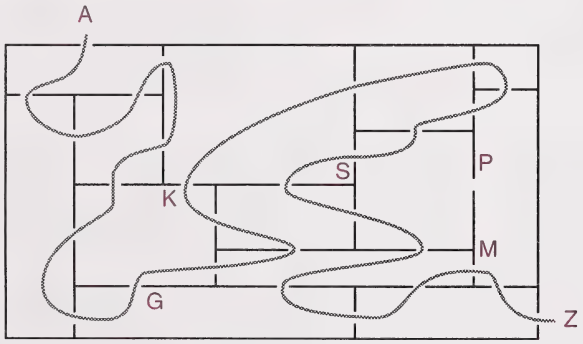
**5.** Use objects to act out the problem.



**6.** The students who move are D, G, and F, as shown.



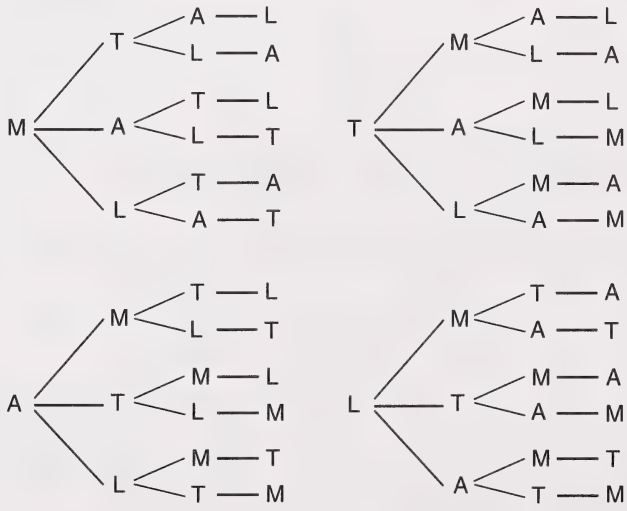

**7.** The class did not pass through door P.




## Section 4: Practice Activity

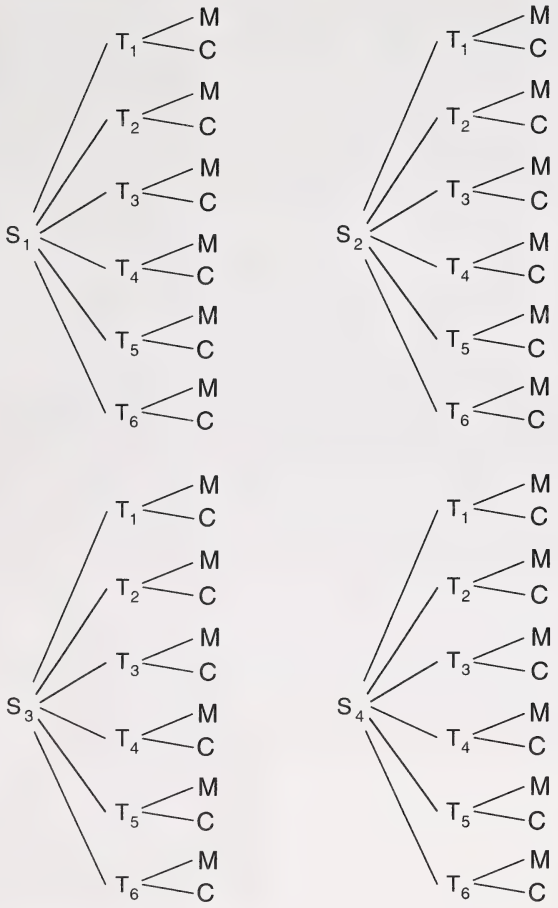
**1.** Make tree diagrams. Use letters to represent the horse posters.

M – Morgan
A – Arabian
L – Lippizaner
T – Tennesee Walking Horse




Stacy could display the posters in 24 different ways.

**2.** Make tree diagrams. Use letters to represent the different sizes, toppings, and cheeses.




There are 48 different pizza combinations.

**3.** Make an organized list of the possible combinations.

| First | Second | Third |
|---|---|---|
| 3 | 0 | 0 |
| 2 | 1 | 1 |
| 1 | 2 | 2 |
| 1 | 0 | 5 |
| 0 | 5 | 0 |
| 0 | 3 | 3 |
| 0 | 1 | 6 |

There are seven possible combinations that will result in a total of 15 points.

**4.** Make an organized list of the possible ways to make a total of $1.47 in stamps.

| 8¢ Stamps | 3¢ Stamps |
|---|---|
| 0 | 49 |
| 3 | 41 |
| 6 | 33 |
| 9 | 25 |
| 12 | 17 |
| 15 | 9 |
| 18 | 1 |

There are seven ways to make a total of $1.47 with 8¢ and 3¢ stamps.

5. Make an organized list.

| Field Goals | Touch-downs |
|---|---|
| 0 | 5 |
| 7 | 2 |

There are only two different ways that a football team can score 35 points if it scores only field goals and converted touchdowns.

6. Make a table.

| Unpacking | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| Assisting | 18 | 17 | 16 | 15 | 14 | 13 | 12 |
| Earned | $3.95 | $4.10 | $4.25 | $4.40 | $4.55 | $4.70 | $4.85 |

↑

Steven assisted 12 customers and unpacked seven boxes that day.

7. Make a table.

| Station Wagons | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| Hatchbacks | 15 | 14 | 13 | 12 | 11 | 10 | 9 |
| Doors | 50 | 52 | 54 | 56 | 58 | 60 | 62 |

↑

There are nine hatchbacks and seven station wagons.

## Section 5: Practice Activity

### Computer Alternative

1. computer checked

### Print Alternative

2. You may use as many guesses as required, but the number of guesses is reduced if you first decide what is a reasonable answer.

   The guesses may be recorded in a table.

| | Left Page | Right Page | Test |
|---|---|---|---|
| Guess 1 | 72 | 73 | $72 \times 73 = 5256$ |
| Guess 2 | 76 | 77 | $76 \times 77 = 5852$ |

The book was opened to page 76 and page 77.

3. You may use as many guesses as required, but the number of guesses is reduced if you first decide what is a reasonable answer.

   The guesses may be recorded in a table.

| | Student's Age | Teacher's Age | Test |
|---|---|---|---|
| Guess 1 | 17 | 25 | $22 \neq 2 \times 14$ |
| Guess 2 | 15 | 27 | $24 = 2 \times 12$ |

The teacher is 27 years old and the student is 15 years old.

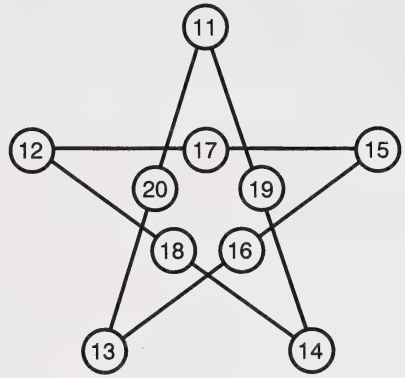
**4.** You may use as many guesses as required.

| | Black Balloons | Red Balloons | White Balloons | Test |
|---|---|---|---|---|
| **Guess 1** | 5 | 4 | 14 | $5 \times 0.65 = \$3.25$<br>$4 \times 0.25 = \$1.00$<br>$14 \times 0.15 = \$2.10$<br>$\$6.35$ |
| **Guess 2** | 4 | 5 | 14 | $4 \times 0.65 = \$2.60$<br>$5 \times 0.25 = \$1.25$<br>$14 \times 0.15 = \$2.10$<br>$\$5.95$ |
| **Guess 3** | 4 | 6 | 13 | $4 \times 0.65 = \$2.60$<br>$6 \times 0.25 = \$1.50$<br>$13 \times 0.15 = \$1.95$<br>$\$6.05$ |
| **Guess 4** | 4 | 7 | 12 | $4 \times 0.65 = \$2.60$<br>$7 \times 0.25 = \$1.75$<br>$12 \times 0.15 = \$1.80$<br>$\$6.15$ |

Barbara bought four black balloons, seven red balloons, and 12 white balloons.

**5.** Use the guess-check-revise method.

You can reduce the number of guesses by realizing that any combination of 16, 17, 18, 19, and 20 will be too large. Therefore, the larger numbers will be on the inside rings.




**6.** Use the guess-check-revise method. You may wish to act out the problem using actual containers or you may prefer to use diagrams.

**Step 1:** Fill the 11-L bowl with rice.

**Step 2:** Pour rice from the 11-L bowl to fill the 6-L bowl. 5 L of rice remain in the 11-L bowl.

**Step 3:** Fill the 2-L bowl with rice, and pour this amount into the 11-L bowl. There is now 7 L of rice in the 11-L bowl.

## Section 6: Practice Activity

### Computer Alternative

1. computer checked

2. computer checked

### Print Alternative

3. Find the pattern using groups of one, two, three, four, five, and six students. You may act this out.

| Number of Students | Number of Introductions |
|---|---|
| 1 | 0 |
| 2 | 1 |
| 3 | 3 |
| 4 | 6 |
| 5 | 10 |
| 6 | 15 |

Pattern

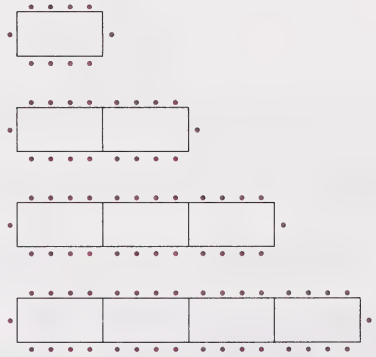
+1
+2
+3
+4
+5

Apply the pattern to a group of 11 students. You may use a calculator.

$15+6+7+8+9+10=55$

In a group of 11 students, 55 introductions were made.

4. Find the pattern using one, two, three, and four tables. You may wish to make diagrams.



| Number of Tables | Number of People |
|---|---|
| 1 | 10 |
| 2 | 18 |
| 3 | 26 |
| 4 | 34 |

Pattern

+8
+8
+8

Apply the pattern to 13 tables. You may use a calculator.

$34+8+8+8+8+8+8+8+8+8=106$

So, 106 people may be seated at 13 tables.

5. Find the pattern by using one, two, three, and four teams. You may wish to make diagrams. Remember each team will play each of the others twice.

| Number of Teams | Number of Games if Each Team Is Played Once | Number of Games if Each Team Is Played Twice | Pattern |
|---|---|---|---|
| 1 | 0 | 0 | |
| 2 | 1 | 2 | +2 |
| 3 | 3 | 6 | +4 |
| 4 | 6 | 12 | +6 |

Apply this pattern to eight teams. You may use a calculator.

$12 + 8 + 10 + 12 + 14 = 56$

For eight teams to play each other twice, 56 games are needed.

## Section 7: Practice Activity

1. Write a simpler problem.

In 1951 Cobequid Junior High won 15 trophies. By 1987 they had won 500 trophies. By how much did the number of trophies increase during this time?

$500 - 15 = 485$

There was an increase of 485 trophies between 1951 and 1987.

Use the same method to solve the original problem.

**Estimate**

$$\begin{aligned} 2\,534\,420\,000 - 309\,465\,000 &\doteq 2.5 \text{ billion} - 300 \text{ million} \\ &\doteq 2500 \text{ billion} - 300 \text{ million} \\ &\doteq 2200 \text{ million or } 2.2 \text{ billion} \end{aligned}$$

The family allowance payments increased by about $2.2 billion.

**Calculation**

$2\,534\,420\,000 - 309\,465\,000 = 2\,224\,955\,000$

The family allowance payments increased by $2 224 955 000.

2. Write a simpler problem.

In 1986 Shawn spent $400 on his music collection. Of this, $40 was spent on compact discs. What percent of the total collection was spent on compact discs?

$\frac{40}{400} = 10\%$

About 10% of the collection was spent on compact discs.

Use the same method to solve the original problem. You will probably wish to use a calculator.

**Estimate**

$$\begin{aligned} \frac{18\,114\,000\,000}{112\,974\,000\,000} &\doteq \frac{18 \text{ billion}}{112 \text{ billion}} \\ &\doteq 16\% \end{aligned}$$

About 16% was spent on engines.

**Calculation**

$$\frac{18\,114\,000\,000}{112\,974\,000\,000} = \frac{18\,114}{112\,974}$$
$$\doteq 16\%$$

About 16% was spent on engines.

**3. a.** Write a simpler problem.

The area of Omar's garden is 40 $m^2$. The area of Raja's garden is 200 $m^2$. How many times larger is Raja's garden?

$$\frac{200}{40} = 5$$

Raja's garden is five times larger.

Use the same method to solve the original problem. You will probably wish to use a calculator.

**Estimate**

$$\frac{22\,402\,000}{9\,976\,000} \doteq \frac{22 \text{ million}}{9 \text{ million}}$$
$$\doteq 2.4$$

The USSR was about 2.4 times the size of Canada.

**Calculation**

$$\frac{22\,402\,000}{9\,976\,000} \doteq 2.2$$

The USSR was about 2.2 times the size of Canada.

**b.** Write a simpler problem.

Ruth's rug is 15 $m^2$. Sarah's carpet is 30 $m^2$. How many times larger is Sarah's rug than Ruth's?

$$\frac{30}{15} = 2$$

Sarah's rug is two times as large.

Use the same method to solve the original problem.

**Estimate**

$$\frac{17\,075\,400}{9\,976\,000} \doteq \frac{17 \text{ million}}{9 \text{ million}}$$
$$\doteq 1.9$$

Russia is about 1.9 times the size of Canada.

**Calculation**

$$\frac{17\,075\,400}{9\,976\,000} \doteq 1.71$$

Russia is about 1.7 times the size of Canada.

## Computer Alternative

**4.** computer checked

**5.** computer checked

**Print Alternative**

**6.** Since several pieces of data are given in the problem, you may find it helpful to organize the information in a table.

| Number of Students | Number of Pets Owned | Total Number of Pets |
|---|---|---|
| 5 | 1 | |
| 8 | 3 | |
| 3 | 0 | |
| ☐ | 2 | |
| | | 51 |

**Step 1:** Find the total number of pets owned by students with one pet.

$5 \times 1 = 5$

So, five pets are owned by students with one pet.

**Step 2:** Find the total number of pets owned by students with three pets.

$8 \times 3 = 24$

So, 24 pets are owned by students with three pets.

**Step 3:** Find the total number of pets owned by students with no pets.

$3 \times 0 = 0$

No pets are owned by students with no pets.

**Step 4:** Find the total number of pets owned by students with two pets.

$$51 - (5 + 24 + 0) = 51 - 29$$
$$= 22$$

So, 22 pets are owned by students with two pets.

**Step 5:** Find the number of students who own only two pets.

$22 \div 2 = 11$

So, 11 students own two pets each.

**7. Step 1:** Find out how much money is earned for working 27 h.

$\$10 \times 27 = \$270$

**Step 2:** Find out how much Anton's father charges for using his snowblower for 27 h.

$\$2 \times 27 = \$54$

**Step 3:** To figure out total profit, subtract how much is owed to Anton's father from how much money was earned in 27 h.

$\$270 - \$54 = \$216$

**Step 4:** Since Anton and his friend split the profit, divide by 2.

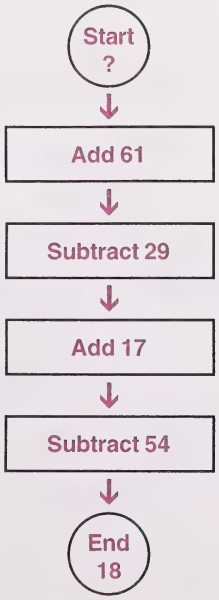
$\$216 \div 2 = \$108$

Anton makes $108.

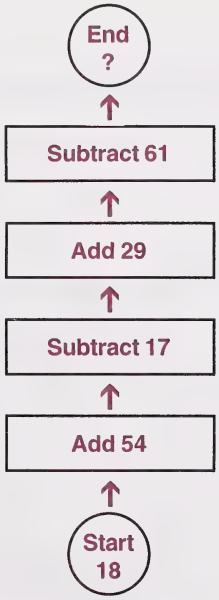
## Section 8: Practice Activity

1. The order of operations in the problem may be shown on a flow chart. To solve the problem, work backwards. Remember to use the opposite operations when working backwards. An inverse flow chart may be helpful.

**Flow Chart**



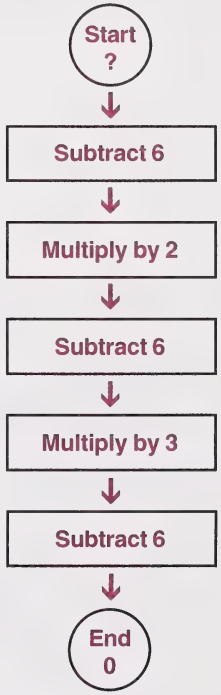

**Inverse Flow Chart**



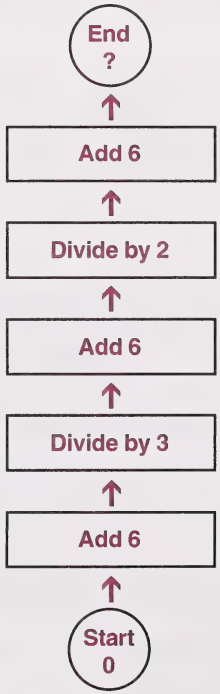

There were 23 cars in the parkade when Bonita started work.

2. The order of operations in the problem may be shown in a flow chart. To solve the problem, work backwards. Remember to use the opposite operations when working backwards. An inverse flow chart may be helpful.

**Flow Chart**




**Inverse Flow Chart**




Ten watermelons were put out on Monday morning.

**3.** The order of operations in the problem can be shown in a flow chart. To solve the problem, work backwards. Remember to use the opposite operations when working backwards. An inverse flow chart may be helpful.

**Flow Chart**

**Inverse Flow Chart**

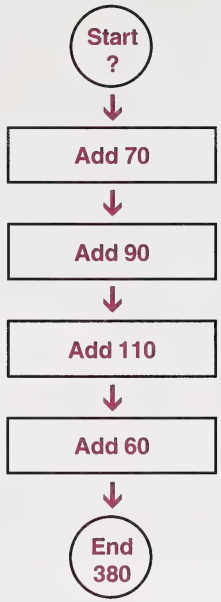
Jeanne began with $78.

**4.** Change all the times to minutes.

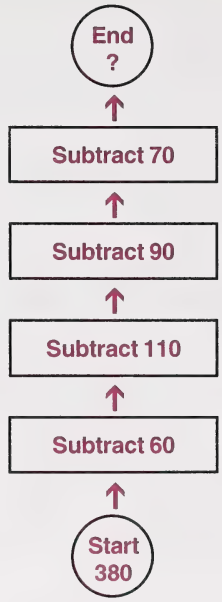
6 h and 20 min = 380 min
1 h and 10 min = 70 min
1.5 h = 90 min
1 h and 50 min = 110 min
1 h = 60 min

The order of operations in the problem can be shown in a flow chart. To solve the problem, work backwards. An inverse flow chart may be helpful.

**Flow Chart**




**Inverse Flow Chart**




Roger must practise for 50 min on the weekend to maintain his average.

## Did You Know?

**1.** 18

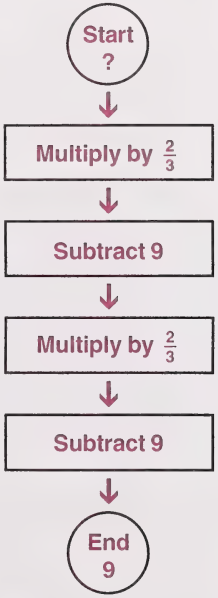
**2.** 11

**3.** 0

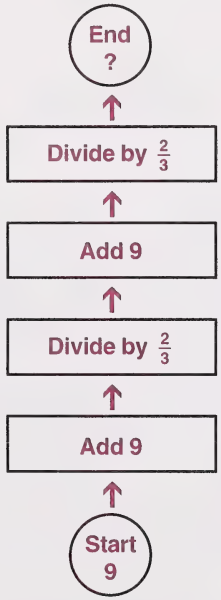
## Section 8: Practice Activity 2

1. The order of operations in the problem can be shown in a flow chart, but be careful – giving away one-third of the pets is the same as multiplying the number of pets by two-thirds. To solve the problem, work backwards. An inverse flow chart may be helpful.

**Flow Chart**




**Inverse Flow Chart**

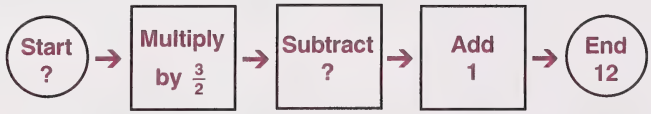



The local shelter had 54 pets at the beginning of the week.

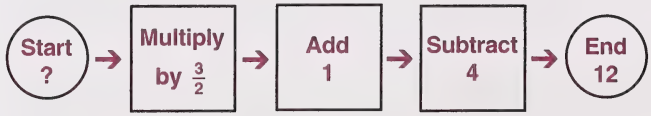
2. There are three sets of actions happening in this problem, so three flow charts are needed. Also be careful – giving people half as much money as they had is the same as multiplying by three-halves. Giving people twice as much as they had is the same as multiplying by 3.

**Sara**

**Barb**




**Shae**



To solve the problem, work backwards. An inverse flow chart may be helpful.

**Sara**

**Barb**

**Shae**

Follow the steps in the inverse flow chart. You may wish to act this out.

**Step 1:** Each has \$12.

Subtract \$3 from Sara $(12-3=9)$. Sara now has \$9.

Subtract \$1 from Barb. Barb has \$11 now $(12-1=11)$.

Add the amount that Sara and Barb lost to Shae $(3+1=4)$. Shae has \$16 now $(12+4=16)$.

**Step 2:** Divide Sara's money by 3. Sara has \$3 now $(9\div 3=3)$.

Subtract \$1 from Shae. Shae has \$15 now $(16-1=15)$.

Add the amount that Sara and Shae lost to Barb's money $(6+1=7)$. Barb has \$18 now $(11+7=18)$.

**Step 3:** Divide Barb's money by $\frac{3}{2}$. Barb has \$12 now $\left(18\div\frac{3}{2}=12\right)$.

Divide Shae's money by $\frac{3}{2}$. Shae has \$10 now $\left(15\div\frac{3}{2}=10\right)$.

Add the amount that Barb and Shae lost to Sara's money $(6+5=11)$. Sara has \$14 now $(3+11=14)$.

So, at the beginning, Barb had \$12, Shae had \$10, and Sara had \$14.

## Section 9: Practice Activity

**1.** The clues indicate the following.

- The grizzly bear and coyote did not finish first. They did not beat the lion.
- The rabbit did not finish first. The coyote beat it.
- The coyote did not finish first. The ostrich was faster.
- The grizzly bear did not finish first. It lagged behind the rabbit.
- The ostrich did not finish first. It was not the fastest.

The clues can be shown on a table.

| | Grizzly | Coyote | Rabbit | Ostrich | Lion |
|---|---|---|---|---|---|
| First | ✗ | ✗ | ✗ | ✗ | |
| Second | | | | | |
| Third | | | | | |
| Fourth | | | | | |
| Fifth | | | | | |

By elimination, it is clear that the lion was first. Now reconsider the clues.

- The rabbit did not finish second. The coyote beat it.
- The coyote did not finish second. The ostrich was faster.
- The grizzly bear did not finish second. It lagged behind the rabbit.

The additional clues can be shown in the table.

| | Grizzly | Coyote | Rabbit | Ostrich | Lion |
|---|---|---|---|---|---|
| First | ✗ | ✗ | ✗ | ✗ | ✔ |
| Second | ✗ | ✗ | ✗ | | ✗ |
| Third | | | | | ✗ |
| Fourth | | | | | ✗ |
| Fifth | | | | | ✗ |

By elimination, it is clear that the ostrich finished second. Now reconsider the clues.

- The rabbit did not finish third. The coyote beat it.
- The grizzly bear did not finish third. It lagged behind the rabbit.

The additional clues can be shown in the table.

| | Grizzly | Coyote | Rabbit | Ostrich | Lion |
|---|---|---|---|---|---|
| First | ✗ | ✗ | ✗ | ✗ | ✔ |
| Second | ✗ | ✗ | ✗ | ✔ | ✗ |
| Third | ✗ | | ✗ | ✗ | ✗ |
| Fourth | | | | ✗ | ✗ |
| Fifth | | | | ✗ | ✗ |

By elimination, it is clear that the coyote finished third. Now reconsider the clues.

- The grizzly bear lagged behind the rabbit.

So, the rabbit was fourth and the grizzly bear was fifth.

### Summary

The order of finishing was the lion, the ostrich, the coyote, the rabbit, and the grizzly bear.

**2.** The clues indicate the following.

- Pedro ate the Bartlett, but not the Seckel.
- Penelope ate the Comice.

Put this information in a table and eliminate the impossibilities.

| | Pedro | Patti | Penelope |
|---|---|---|---|
| **Anjou** | | | |
| **Bosc** | | | |
| **Bartlett** | ✔ | ✗ | ✗ |
| **Clapp's Favorite** | | | |
| **Comice** | ✗ | ✗ | ✔ |
| **Seckel** | ✗ | | |

Now consider the other clues.

- One person ate two pears beginning with the same letter. That person was one of the girls.

- Two pairs of pears begin with the same letter – the Bartlett and the Bosc, and the Clapp's Favorite and the Comice.

Use logical reasoning. Since Pedro ate the Bartlett, it is clear that a girl ate both the Clapp's Favorite and the Comice. This means that Penelope must have eaten the Comice and the Clapp's Favorite, and Pedro could not have eaten the Bosc.

Fill in the table with the additional clues.

| | Pedro | Patti | Penelope |
|---|---|---|---|
| **Anjou** | | | ✗ |
| **Bosc** | ✗ | | ✗ |
| **Bartlett** | ✔ | ✗ | ✗ |
| **Clapp's Favorite** | ✗ | ✗ | ✔ |
| **Comice** | ✗ | ✗ | ✔ |
| **Seckel** | ✗ | | ✗ |

It is now clear Pedro ate the Bartlett and the Anjou. This means that Patti ate the Bosc and the Seckel.

**Summary**

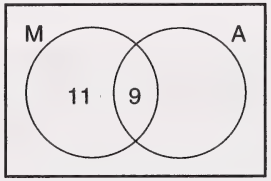
Pedro ate the Bartlett and the Anjou, Patti ate the Bosc and the Seckel, and Penelope ate the Comice and the Clapp's Favorite.

## Section 10: Practice Activity 1

1. The clues indicate the following.

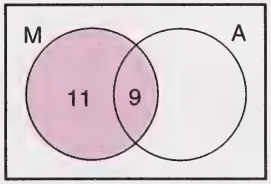
    - There are 32 students.
    - Eleven students take only music.
    - Nine students take both music and art.
    - All students take either music or art, or both.

Make a Venn diagram.




Calculate the number of students who take music.

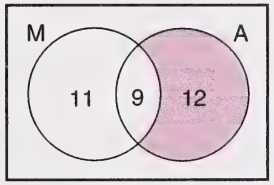
$11 + 9 = 20$




So, 11 students take only music.

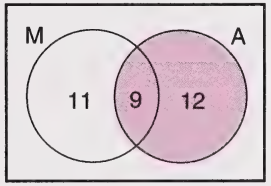
Calculate the number of students who take only art.

$32 - 20 = 12$




Calculate the number of students who are enrolled in art.

$9 + 12 = 21$

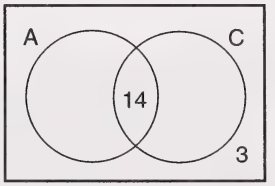



So, 21 students are enrolled in art.

**2.** The clues indicate the following.

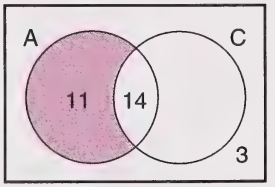
- Twenty-five students brought an apple in their lunch.
- Twenty students brought a cookie.
- Fourteen students brought both an apple and a cookie.
- Three students brought neither an apple nor a cookie.

Make a Venn diagram.




Calculate how many students brought only an apple.

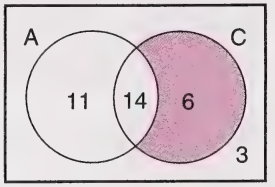
$25 - 14 = 11$



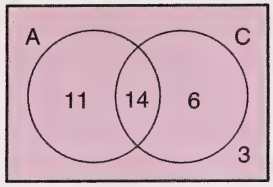

So, 11 students brought only an apple.

Calculate how many students brought only a cookie.

$20 - 14 = 6$



Calculate the total number of students.




$11+14+6+3=34$

There are 34 students altogether.

## Section 10: Practice Activity 2

**1. a.** $14+4+3+5=26$

So, 26 students play tennis.

**b.** There are 14 students who play tennis, but not soccer and volleyball.

**c.** $21+4+3+2=30$

So, 30 students play soccer.

**d.** There are 21 studetns who play soccer, but also play tennis and volleyball.

**e.** $5+3+2+10=20$

So, 20 students play volleyball.

**f.** There are 10 students who play volleyball, but not tennis and soccer.

**g.** $4+3=7$

So, 7 students play tennis and soccer.

**h.** $5+3=8$

So, 8 students play tennis and volleyball.

**i.** $3+2=5$

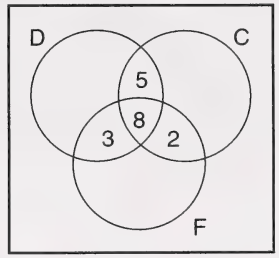
So, 5 students play soccer and volleyball.

**j.** There are 3 students who play all three sports.

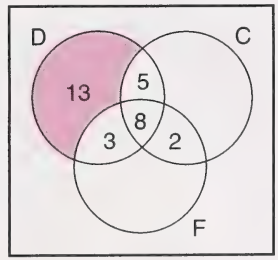
**2.** The clues indicate the following.

- There are 50 families altogether.
- Twenty-five families have cats.
- Twenty-seven families have dogs.
- Eighteen families have tropical fish.
- Five families have only cats and dogs.
- Three families have only dogs and tropical fish.
- Two families have only cats and fish.
- Eight families have all three kinds of pets.

Make a Venn diagram.




Calculate the number of families that have only dogs.



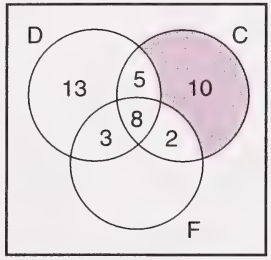

$27-(5+8+3)=27-14$

$=13$

Calculate the number of families that have only cats.

$25-(5+8+2)=25-15$

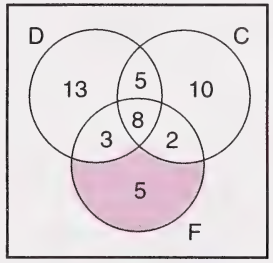
$=10$




Calculate the number of families that have only tropical fish.

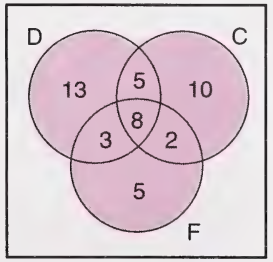
$18-(3+8+2)=18-13$

$=5$



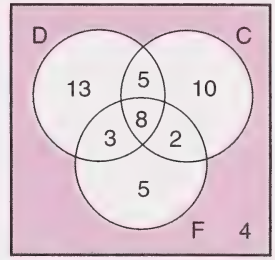

Calculate the number of families who have either dogs, cats, or fish.

$13+5+8+3+10+2+5=46$




Calculate the number of families who do not have any of these pets.

$50-46=4$




So, four families do not have a dog, cat, or tropical fish.

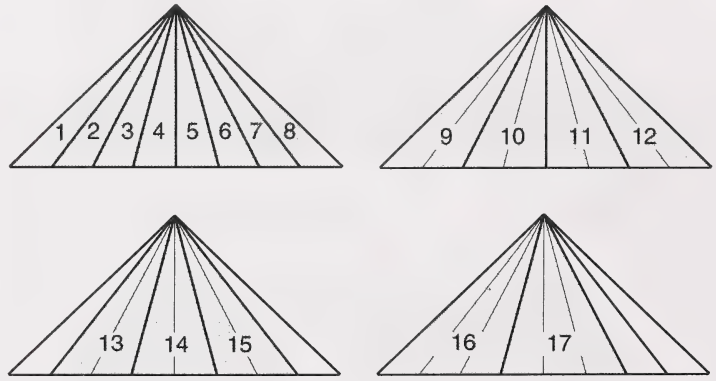
## Section 11: Practice Activity 1

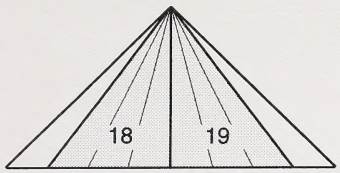
### Computer Alternative

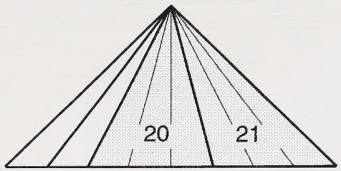
**1.** computer checked

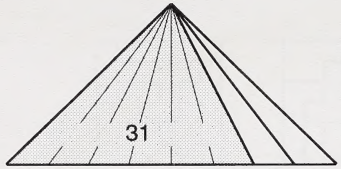
### Print Alternative

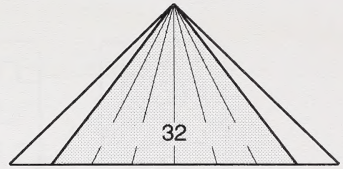
**2.** **Method 1:** Make an organized list of the triangles.

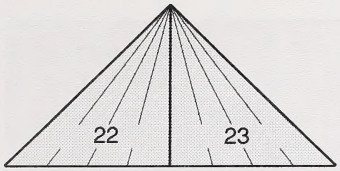


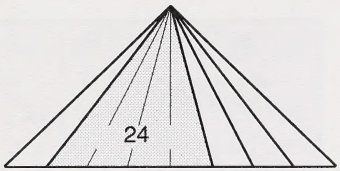


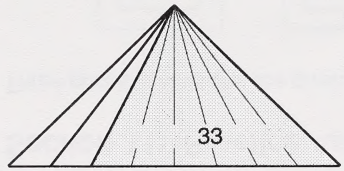



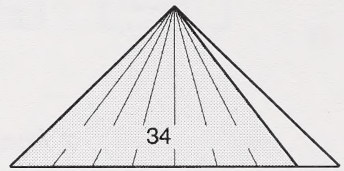



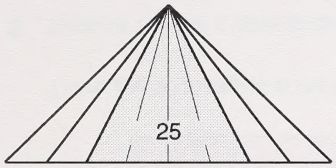



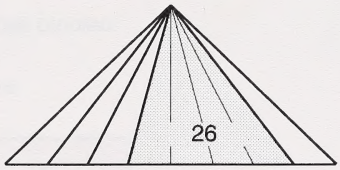



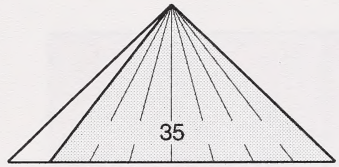



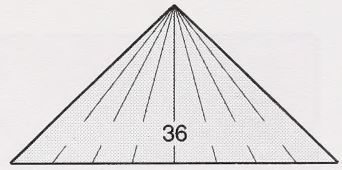



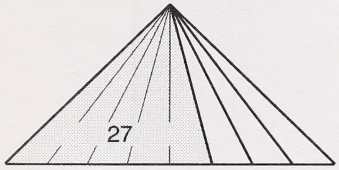



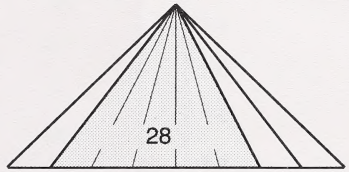



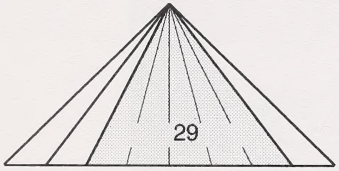



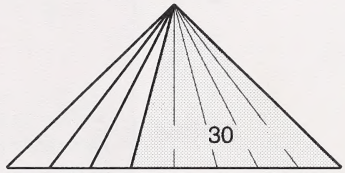



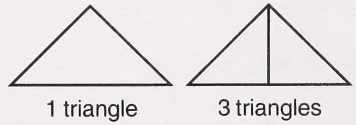



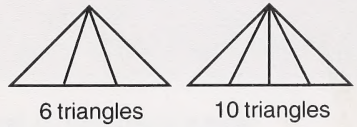




**Method 2:** Find a pattern and apply it.



| Number of Subdivisions | Number of Triangles | Pattern |
|---|---|---|
| 0 | 1 | |
| 1 | 3 | +2 |
| 2 | 6 | +3 |
| 3 | 10 | +4 |
| 4 | 15 | +5 |
| 5 | 21 | +6 |
| 6 | 28 | +7 |
| 7 | 36 | +8 |

There are 36 triangles in the figure.

3. **Method 1:** Use a table to solve this problem.

| Four-engine Planes | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Two-engine Planes | 26 | 25 | 24 | 23 | 22 | 21 | 20 | 19 |
| Total Engines | 56 | 58 | 60 | 62 | 64 | 66 | 68 | 70 |

↑

Meredith saw eight four-engine planes.

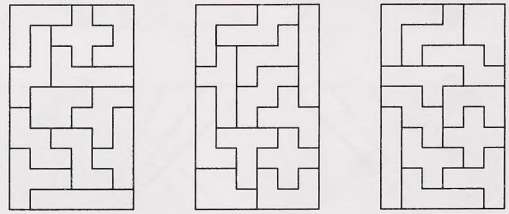
**Method 2:** Use guessing, checking, and revising to solve the problem. You may use as many guesses as required. The guesses may be recorded in a table.

| Guess | Two-engine Planes | Four-engine Planes | Test |
|---|---|---|---|
| 1 | 10 | 17 | $10 \times 2 = 20$<br>$17 \times 4 = 68$<br>88 |
| 2 | 13 | 14 | $13 \times 2 = 26$<br>$14 \times 4 = 56$<br>82 |
| 3 | 19 | 8 | $19 \times 2 = 38$<br>$8 \times 4 = 32$<br>70 |

Meredith saw eight four-engine planes.

## Section 11: Practice Activity 2

Three possible solutions are given.



3 3286 50163 4782

Mathematics 9

Student Module Booklet

Module 1

L.R.D.C.
Producer

1993